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EXAMINING MATHEMATICAL TEXTS WRITTEN BY PROSPECTIVE TEACHERS: CLARIFYING LINGUISTIC FACTORS

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ABSTRACT

The present study aims to examine the mathematical language use of primary prospective teachers through the mathematical texts they write. Forty-seven 3rd-year prospective teachers studying in the Department of Mathematics Education at a university participated in the study. The prospective teachers were asked to solve 4 open-ended mathematics problems, in which 3 of them were taken from the Programme for International Student Assessment (PISA) 2012. The data were analyzed through the content analysis method. The main findings of the study illustrate that the language used by the prospective teachers in the mathematical texts they have written were incomplete or incorrect in logical connective use, equal, congruent, approximate symbols, time measurement expressions, rational numbers, abbreviation, division algorithm, ratio-proportion representation and the presentation of solutions. The errors and ambiguity in participants' proportion

Keywords: *Mathematics education, Mathematical language, Mathematical texts, Prospective teachers*

INTRODUCTION

Part of learning mathematics is to learn the use of mathematics-related codes and symbol systems, namely language (Schleppegrell, 2007). That is because mathematics is dependent on language (Kim et al., 2012). Mathematical codes and systems formulate the conceptualization and development of mathematics and enable mathematical ideas to be conveyed comprehensibly (Sastre et al., 2013). Learning to write and speak mathematics like a mathematician is not about simply learning the jargon, but also learning to be a mathematician (Rowland, 2001). Despite this, linguistic processes have generally been overlooked in mathematics teaching. However, various problems may arise if these processes are not carefully organized (Morin & Franks, 2009). That is because the connections formed by students between language and written symbols can be different from those of adults (Muzvehe & Capraro, 2012). What should be done in this case? Perhaps more so compared to other disciplines, the structuring of knowledge in mathematics depends on teachers (Schleppegrell, 2007). Muzvehe & Capraro (2012) stated that the language used by teachers create or reinforce implicit images. Similarly, Rowland (1995) stated that the words used by students may originate from the ideas imposed on them by their teachers through the language they use, whether wittingly or unwittingly. Similarly, Schleppegrell (2007) emphasizes the key role of teachers in learning to confer symbols, diagrams, and mathematical words. It is stated that even in the pre-school period, teachers serve as role-models for their students in terms of the acquisition of academic language, which supports the mathematical development of students (Brunner et al., 2006, cited in Michel et al., 2014). It is also stated that teachers should be aware of this significant role regarding this acquisition and possess the necessary skills (Keuch

& Brandt, 2018; Michel et al., 2014). Teacher training institutions are responsible for enabling prospective teachers to acquire the necessary knowledge, skills, and competence related to mathematical language and possess the necessary skills. In this context, the present study aims to examine the mathematical language use of primary prospective mathematics teachers through the mathematical texts they write. In line with this purpose, the answer to the question "How do prospective teachers use mathematical language in the mathematical texts they write?" was sought. The mathematical texts written by participants were examined in terms of mathematical convenience and accuracy as well as appropriateness for mathematics teaching.

Mathematical Language

There are various challenges in coming up with a detailed description and definition of mathematical language and its content (Morgan, 1998). For a certain time period, mathematical language was regarded as a model that individuals had to comply with and its characteristic features consisted of syntax, semantics, and lexicon. In recent years, studies on mathematical language have been taking the important features regarding its use into consideration as well (Morgan et al., 2005). That is because words, terms or texts have various meanings, functions and purposes depending on the practices they are embedded in (Moschkovich, 2007; Rowland, 1995). At this point, Morgan et al. (2005) assumes that conducting an in-depth investigation of mathematical language and all its aspects is impossible without considering the linguistic systems involving the written and spoken (verbal) language adapted to mathematics at all levels, symbolic notations, visual representations and even gestures and facial expressions.

Written language, symbolic notations, visual representations, spoken language, gestures and facial expressions work in conjunction to structure the meaning in the interaction between the student and teacher in the classroom. Writing and speaking are ways to visualize mathematical thinking and they enable the learner to develop a personal language (Whitin & Whitin, 2000). Schleppegrell (2007), who emphasizes the distinction between verbal, written and symbolic language, stated that verbal language reinforces the complexities in the language used in the classroom. That is because verbal language is unable to grasp relationships as well as written and symbolic language (Schleppegrell, 2007). Writing shares many aspects with speaking, however, writing has more unique features such as creating a record of thoughts to allow for analyzing and reflecting on them (Whitin & Whitin, 2000).

Mathematical Texts

In terms of the importance of writing, it is necessary to answer the questions of how mathematical writing should be performed or what the features of a mathematical text should be. Morgan (1998), who examined the use of language in mathematical texts in terms of ideological and social aspects, discusses the concept of mathematics register, which was introduced by Halliday, an author who provided definitions regarding the general properties of mathematical texts, in the 1974 UNESCO Interactions Between Linguistics and Mathematical Education Symposium. Halliday (1974) defined mathematics register as follows:

> A set of meanings that is in accordance with the special functions of language and includes forms of discussion and styles of meaning along with the structures and words that represent these meanings (p. 65).

Therefore, the linguistic properties that enable the inclusion of a text in the mathematical context includes its words, symbolic context, grammatical structure, and the forms of discussion used (Morgan, 1998). At this point, the process of recognizing mathematical texts requires recognizing and distinguishing different types of language (Rowland, 2001). It is not sufficient to know only the mathematical expressions such as "less", "more" and "up to", therefore, it is necessary to examine the language patterns formed by these expressions and how they structure mathematical concepts (Schleppegrell, 2007). Rowland (2001) provides various categories of language that clarifies this structuring. For example, the statement "A tetrahedron has 4 faces, any two of which intersect on one edge" includes categories such as explicit logical language ("each-any"), implicit logical language ("one" has the significance of "each"), math-specific words ("tetrahedron"), words originating from natural language (having, intersecting), and words originating from natural language and redefined for mathematical purposes (such as "face") (Rowland, 2001, p. 186).

Mathematical Language and Mathematics Teachers

According to Kim et al. (2012), teachers should be aware of the linguistic features of discourses that support learning. Teachers are also expected to be mindful of the language used by them and their students, and to be able to analyze it. This is so that they are able to implement language in a positive way and avoid linguistic barriers (Morin & Franks, 2009). Furthermore, Rowland (2001) emphasizes that linguistic awareness is part of the occupational knowledge of teachers. Teachers' use of linguistic sources to include students in classroom interaction may provide opportunities for dialogues that can affect the performance of students or prevent them. That is because teachers can change the dynamics of communication in classroom debates through the use of language (Mesa & Chang, 2010). Adams (2003) states that teachers can lead their students to mathematics register by means of helping them to recognize and implement mathematical language in defining and explaining concepts rather than using informal language, forming connections between the daily and mathematical meanings of words, particularly vague terms, homonymic words and similar-sounding words, therefore eliminating the misconceptions resulting from semantic uncertainties and ambiguities (Rowland, 2001), as well as clearly evaluating students' skills of using mathematical language (Schleppegrell, 2007).

In the literature, there are studies which integrate writing to mathematics education for assessing students' mathematical understanding (Adu-Gyamfi et al., 2010), for developing problem solving skills (Bicer et al., 2013), and learning mathematics (Baxter, 2008; Ntenza, 2006; Pugalee, 2001). These studies suggest the mathematics teachers to use writing in teaching mathematics. In this point, the quality of mathematical writing of teachers or prospective teachers becomes important. When the related literature is examined, it can be said that the use of mathematical language by teachers and prospective teachers is inadequate (Kabael, 2012; Michel et al., 2014; Raiker, 2002). According to Raiker (2002), although teachers use the words suggested by a mathematical dictionary, they are not aware of the importance of the words they use and they do not plan the explanation, repetition and instruction of the meaning of these words. They are unable to distinguish between the differentiating purposes of language and, therefore, fail to activate students cognitively in the teaching of important words (Raiker, 2002). According to Kabael (2012), the skills of primary prospective mathematics teachers to write in mathematical language and read this language with comprehension is not adequate even to evaluate the meaning of the mathematical sentences written by themselves.

METHODOLOGY

The present study, which was designed as qualitative due to its purpose and the nature of the research problem, is a case study as it aims to reveal the pre-existing mathematical language use of prospective teachers without any intervention. The case of the study is primary prospective mathematics teachers while the unit of analysis is the mathematical language use of participants.

Study Group

Participants consist of third-year students studying in the primary mathematics education department at a state university in Turkey. The criterion taken into consideration in the selection of participants, which was carried out using the purposeful criterion sampling method, is for the students to have taken particularly the General Mathematics, Analysis I, II and Discrete Mathematics courses. In these courses, the basic mathematical concepts, operations, and symbols are examined, symbolic logic is used, mathematical concepts are discussed and converted to each other with multiple representations, relations are examined and the basic skills for writing mathematics are acquired. Therefore, considering that the mathematical language use of the prospective teachers may improve at least to some extent in the first three years of their undergraduate education, participants were selected among 3rd-year students. In this context, 47 prospective teachers voluntarily participated in the present study.

Data Collection

In the data collection phase, the prospective teachers were provided with a semi-structured form requiring the solution of 4 open-ended mathematics problems taken from PISA. In the form, it was stated it will be examined how they solved these questions as a prospective teacher, and they were asked to solve the given problems, present their solutions mathematically, as well as present different solutions, if any. The prospective teachers were informed that the problems were prepared at the level of 15-year-old students. Participants solved the problems without any intervention and stated their solutions in written form.

Two out of the seven mathematical competencies included within the framework of mathematical literacy, which is the theoretical basis of PISA 2012 as an assessment of the mathematical literacy skills of 15-year-old students in addition to the fields of Reading and Science, consists of Communication and the Use of Symbolic, Formal and Technical Language and Operations (Organization for Economic Co-operation & Development (OECD) 2013). In this context, it was thought that the problems taken from PISA 2012, which adopts a framework where the use of mathematical language is effectively emphasized (mathematical literacy), would provide sufficient opportunities to examine the mathematical language use of the prospective teachers. In order to examine participants', use of mathematical language extensively, it was ensured that the problems included different learning domains and mathematical skills. In this context, the problems that requires explanation through estimations and approximate values on rational numbers and operations with rational numbers, on geometry, on length, time and speed, and the area measurement. Selected PISA problems are Climbing Mount Fuji, Revolving door, Oil spill (see OECD, 2012).

Data Analysis

To investigate participants' use of mathematical language, the data were analyzed utilizing the content analysis method. In order to reveal the different types of language and language patterns in the mathematical texts, the language categories set out by Rowland (2001) consisting of explicit/implicit language of logic, math-specific words, and words originating from natural language were used. The mathematical texts written by prospective teachers were examined within the framework of logical language, mathematical symbols and words originating from natural language. In other words, the attribute of language used in mathematical texts were examined in terms of the effectiveness and accuracy of the use of these elements of mathematical language. The mathematical texts written by participants were read several times, evaluated based on categories, and the patterns regarding participants' use of language in their written and symbolic expressions and drawings were revealed. In addition to their mathematical convenience and accuracy, the mathematical texts were also examined and interpreted in terms of their quality for mathematics teaching.

Validity and Reliability of the Study

Participants were informed about the purpose of the study and its voluntary nature, and that their personal information would be kept confidential throughout the study, especially in the analysis and interpretation of data. It is thought that this way, participants presented their solutions based on their knowledge, without manipulation and in a voluntary manner. Peer review was performed as the external control mechanism (Lincoln & Guba, 1985) and expert opinion was referred to for the reliability of data analysis. The correlation coefficient between researcher and expert was calculated as 0.94. The confidentiality of participants' personal information and the use of peer review in the analysis and interpretation of data supported the objectivity (Miles & Huberman, 1994, p. 278). In order to ensure the transferability of the findings and results of the study, the Methods, Findings and Interpretation sections were detailed and it was aimed to enable readers to make their own settings when necessary (Miles & Huberman, 1994).

RESULTS

In the study, the findings consisted of the titles of explicit logical language, symbolic representation, and the presentation of solutions. Table 1 summarizes main findings of mathematical language use of prospective teachers and frequency. Explanations and interpretations on these findings are included in the ongoing subtitles.

Table 1
Mathematical Language Use of Prospective Teachers, n (%)

Writing in Mathematics		n (%)
Explicit logical language	Incorrect or improper uses of logical connectives	33 (%70)
Symbolic representation	Incorrect or improper uses of equals, congruent and approximate symbols	8 (%17)
	Ignoring numerator, denominator, and fraction bar of a rational number	7 (%15)
	Improper application of abbreviation or division algorithm	5 (%11)
	Different representations of ratio-proportion	28 (%59)
Presentation of the solution	Unclear, disconnected and procedural solutions	18 (%38)

Explicit Logical Language

It was observed that participants used the symbols of logical language such as material conditional (→ , if ...then); logical consequence (⟹); logical biconditional (iff) (⇔); some of the relation symbols (such as $\sim, \simeq, \cong, =$). However, there were incorrect or improper uses of these symbols by participants. It can be said that participants used the logical connective "if …then" (→) symbol in every improper case and that almost all of participants used this symbol improperly.

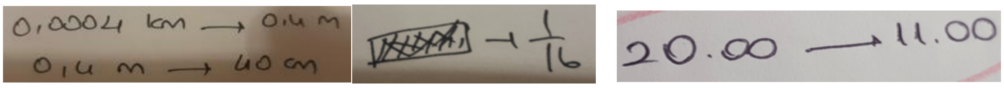


Figure 1. The use of the "if … then" (→) connective

As shown in Figure 1, it was observed that participants used the connective to list the data related to the problems or the relationships between the quantitative variables in the problems, to state related objects, to explain meaning of given information, and to list and model results by means of equals. For example, participants used "6.5 cm → 65 km". It can be observed that this statement, which stands for "If it is 6.5 cm, then it is 65 km", is not logical. However, there are certain correct uses in the solution of equations, such as "1 dose → 1/16 level".

Logical consequence (⟹) connective states that with p and q as propositions, if p→q is a tautology, then the p, q proposition is logically entailed (Arıkan & Halıcıoğlu, 2013). However, prospective teachers used this connective with the aim of regulating data in improper cases. For example, there were uses such as "Climbing ⟹ 3/2 x t= 9", "Perigonally ⟹ 3a=360°", "Arc length ⟹ $\frac{2\Pi.100.120}{360}$" or "Step counter ⟹ 22.500". On the other hand, it was observed that one of participants correctly stated the approximate value of a number.
When the biconditional connective "p if and only if q" is true for each truth value, i.e. a tautology, $p \leftrightarrow q$ is a logical equivalence. Only 3 students used the logical equivalence (⇔) symbol properly. It was observed that one participant used the symbol inconsistently. For example, the student used it properly, but unable to do so afterwards. In addition to this, it was observed that participants did not use the

coordinating logical conjunction "and" (∧), the logical disjunction "or" (∨), or the biconditional connective (↔).

Symbolic Representation

Under this heading, findings related to the symbolic representations of equals, congruent and approximate symbols, rational numbers, abbreviation, division algorithm and ratio-proportion, and the unconventional symbolic representations of participants were included. Participants used the equals symbol in improper contexts. For example, they used the symbol (in the form of “diameter=200 cm”) to define unknown values (Ayşe=a) or variables (landing time=y). When used in the right context, it was observed that there were errors in symbolic representation and writing symbols. For example, as shown in Figure 2, in operations with rational numbers, participants used the symbol to describe only the numerator of rational numbers and not the rational number’s itself, or in a way that the described equation is uncertain. Therefore, the equals symbol was used incorrectly in terms of semantics and syntax.

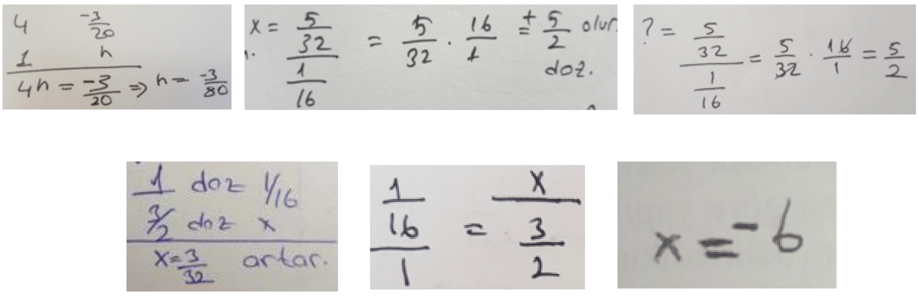


Figure 2. The use of the equals symbol (=)

It can be said that these uses may be one of the reasons behind the misconceptions of students regarding the equals symbol. On the other hand, prospective teachers also possess similar misconceptions. Therefore, these writing mistakes may be due to the inadequacy of participants' comprehension of the concept. When the first five figures are examined, it is understood that participants' conceptions regarding the concept of rational numbers should be examined in the context of numerator, denominator and fraction bar. It can be said that the writings shown in Figure 2 demonstrate an immature content knowledge.

Problems require solvers to estimate results and express approximate values. In this context, when participants' use of symbols to express estimation was examined, it was observed that the term "round up" or the symbol ~ (congruent) were used (See Figure 3 (a)). Majority of participants used the equals symbol for approximate values.

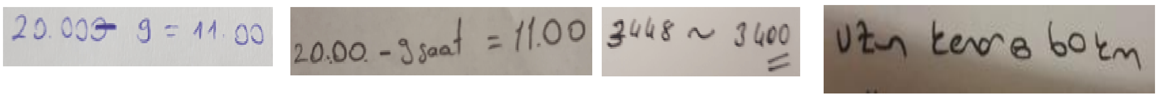


Figure 3. Improper representations

As shown in Figure 3 (b) and (c), it can be said that participants used certain time expressions incorrectly. Some of participants used the symbol ":" in defining the unknown, which is a correct way. However, one of participants changed the symbol as shown in Figure 3 (d) and used this in substitution for the equals symbol. However, the representation will still be incorrect even if the equals symbol is used.

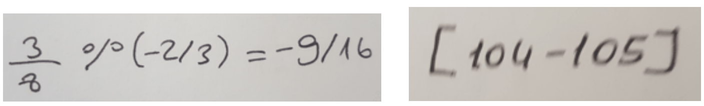


Figure 4. Different and improper representations

Similarly, it was observed that the conventional division symbol was used in a similar way to the symbol % (percentage) and that the closed interval was used incorrectly (See Figure 4). In the problem Revolving Door 2, participant aimed to present an interval for the solution but represented the resulting solution interval as shown in Figure 4. It can be said that such errors in the use of symbols are problematic in terms of student comprehension.

Additionally, in the symbolic interpretation of the conventional division algorithm, which is based on iterative subtraction, participants did not pay much attention to digits in expressing remainder:

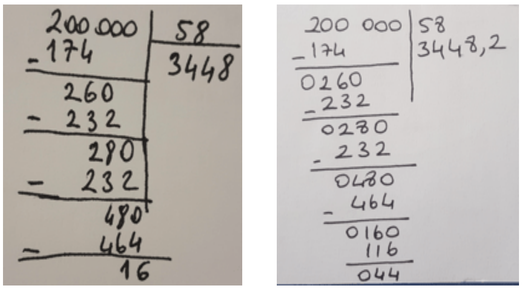


Figure 5. Division algorithms

It can be said that participants performed abbreviation in operations with rational numbers in a way that can be considered as symbolically primitive or erroneous. For example, they expressed abbreviation as "removing zero". Participants could have demonstrated this operation with exponential expressions instead of crossing off the zeros. Although this method is not erroneous, it can be considered as an immature approach that can lead to misconceptions in students regarding the concept of digits.

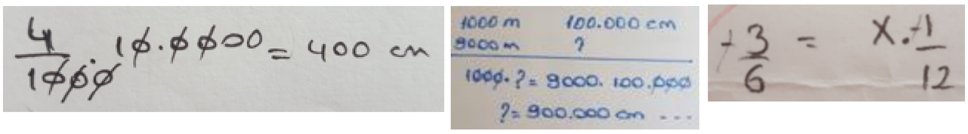


Figure 6. Abbreviation representations

The error made by participant as shown in Figure 6 is a mistake that can be observed in students as a result of the said approach. It should be noted that participant erased the zeros starting from the hundred thousands digit instead of the units digit and expressed the number 100.000 as 10.0000. It is clear that such mistakes will be very risky in teaching. In terms of the potential to cause misconceptions, another one of participants expressed the number 3448.27 as 3,448,27. It was also observed that a participant produced a positive number from the division of two negative rational numbers by drawing a line on the symbols (Figure 6). It can be considered that participants had limited uses for the representation of negative rational numbers. It was observed that all but three of participants used the negative sign only for the numerator. This is thought to be due to the fact that the number is written like this in the problem text. However, the prospective teachers would be expected to be aware of the equation of $-\frac{a}{b}=\frac{(-a)}{b}=\frac{a}{(-b)}$ and use it.

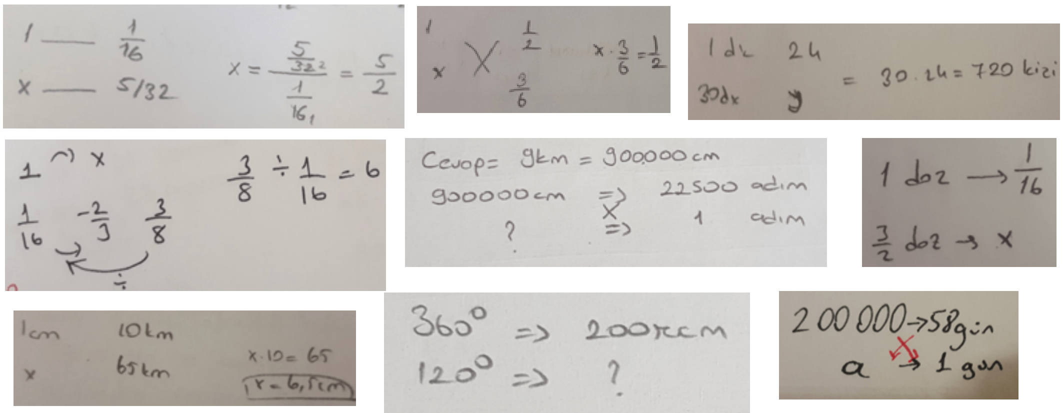


Figure 7. Proportion representations

It was observed that participants used incomplete or unfamiliar representations in proportion and solving it. Participants produced improper representations in all three steps in determining and comparing the equivalence of proportions and solving proportions. They used proportion as the relation between four terms equal to the ratio of the first to the second and the third to the fourth. The proper representation for proportion may be a/b = c/d; a:b=c:d; axd=bxc (b and d non-zero), however, these representations are not clear in the solutions of participants. When Figure 7 is examined, it can be said that the proportion established by participants is only directed towards carrying out the operation and that the meaning of proportion, the terms and the relation between the terms are unclear from the mathematical texts written by the prospective teachers.

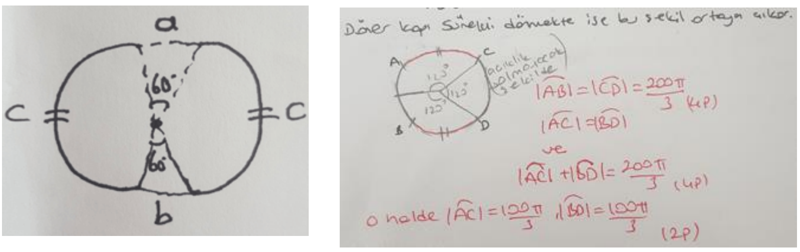
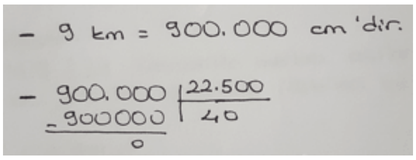


Figure 8. Geometric figures and representations

In general, it was observed that participants did not use unit in their solutions. For example, they overlooked the use of the degree symbol or the use of other units when mentioning angle measurement. Additionally, majority of participants used variables, unknowns and labels in formulas and operations without defining them. For example, they did not define the letters x, a or b. They used the $2\pi r$ formula, however, none of participants defined "r". This may be due to the conventional circle radius symbol of r. As shown in Figure 8, it was observed that some of participants drew shapes that did not resemble circle or circular region in the drawings they produced in the Revolving Door problem. In the question involving the arc of circumference within the framework of this problem, it was observed that only one participant expressed arc length, equations and relations using symbols and correctly (Figure 8). Participant's use of the equals symbol is erroneous in terms of rational numbers, however, the use of connectives is appropriate and therefore, her solution is comprehensible.

Presentation of the Solution

When the presentations of participants' solutions are examined, the complexity of the order of steps is notable. Additionally, the solution of majority of participants included only operations, excluded conceptual explanations, was difficult to comprehend, the order of operations was inaccurate, and not suitable for distinguishing between operations and understanding the order. The order of operations is unclear and disconnected. The conceptual basis and underlying ideas behind the solutions of participants cannot be understood from the mathematical texts they wrote. It can be said that one of the main reasons for this is the errors or lacking in the logical conjunction use of participants. In addition to the errors in conjunction use, the linear structure of writing orders makes them more difficult to comprehend. It was observed that one of participants used the "- (dash)" symbol as the bullet to indicate the order of operations:



or - 50x 50= 2500 km^2

Figure 11. Improper use of the dash symbol

This symbol might be problematic in terms of student comprehension, in addition to being improper in terms of mathematical language. Additionally, the lack of conjunction use, erroneous conjunction uses and the use of equals as a conjunction makes it more difficult to comprehend the mathematical texts written by participants (See Figure 12). Figure 12 exemplifies errors in the presenting the solution.

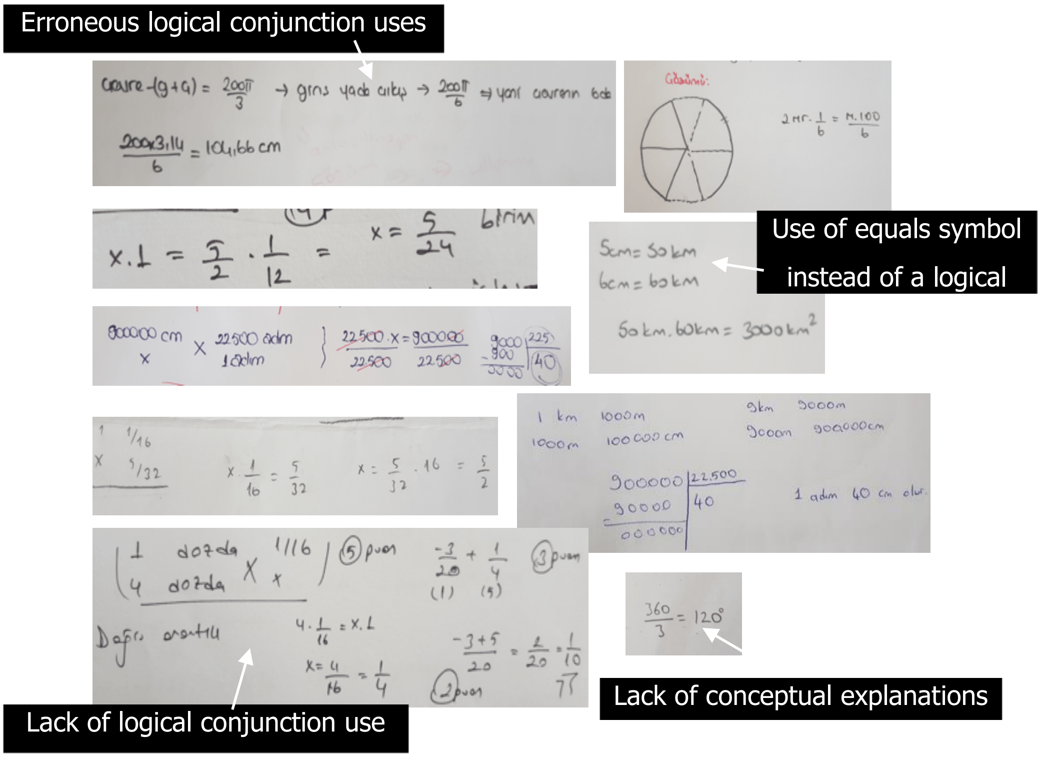


Figure 12. Presentation of the solution

In the solutions; determination of numbers, application of the operation and emphasis of the result are notable. It can be said that participants followed a sequence of steps including "determine the numbers, perform the operation, finish".

DISCUSSION

The mathematical texts written by the prospective teachers were examined and categories of explicit logical language, symbolic representation, and the presentation of solutions were obtained. Therefore, the view that mathematical language includes more than a customized vocabulary (Morgan, 2005) is supported. Within the framework of these categories, it can be said that the language used by participants was lacking and erroneous in terms of logical conjunction use, equal, congruent, and approximate symbols, time measurement expressions, rational numbers, abbreviation, division algorithm, ratio-proportion representation, operation with rational numbers and the presentation of solutions. Similarly, in the study conducted by Saenz (2009), it was reported that common difficulties experienced by prospective teachers in the solution of PISA problems were communication, symbolization and the use of symbolic, formal and technical language and operations.

In regard to the division operation, it was observed that participants used the conventional division algorithm based on repetitive subtraction. However, participants used this algorithm without paying attention to the digit value of remainder. Additionally, it was observed that participants expressed numbers using commas without paying attention to digit values. Similarly, some of participants made mistakes when expressing 5 or 6-digit numbers, even though writing 5 or 6-digit numbers is a subject of elementary mathematics. Therefore, it is thought that participants' knowledge of basic numbers is insufficient. It was also observed that participants expressed the abbreviation they performed in rational numbers or the division algorithm as "removing zero" and performed the operation as such. Furthermore, participants did not pay attention to the digit values of the zeros they removed, or they abbreviated them by crossing them off. The underlying reason may be that their pre-existing operational knowledge may be not based on a conceptual basis and that their rote usage became deformed over time. This reveals the long-term trouble of rote knowledge in mathematics, particularly for mathematics teachers.

It was determined that participants' use of language was improper in the demonstration of rational numbers, negative rational numbers and operations with rational numbers. Rational numbers and the operations performed with them are important especially in solving equations and algebraic expressions. However, based on the findings obtained in the present study, it can be concluded that participants were inadequate in terms of expressing the chain of steps in the arithmetic operations they performed with rational numbers. Behr et al. (1983) stated that although there was more emphasis on operations with rational numbers and their algorithms in the schools, the performance of students in operations with rational numbers was surprisingly low (Carpenter et al., 1980; Behr et al., 1983). On the other hand, when the mathematical language used by the prospective teachers is examined, this situation becomes less surprising and more predictable. Putra (2018) states that the representation and use of rational numbers directly affects the operations performed with rational numbers by students.

The importance of rational numbers in the concept of proportion and in performing the mental actions related to proportion is emphasized (Vanhille & Baroody, 2002). The participants used rational numbers and proportion together. It was observed that participants produced different representations in stating and comparing the equivalence of proportions, and that they used different logical conjunctions. However, the majority of these were improper. There were improper uses in terms of the equals symbol, the representation of numbers and the use of conjunctions that enable to follow the operation steps when solving the proportions. Prospective teachers who use the equals symbol in inappropriate ways, such as equating the denominator may overlook the idea that the equals symbol refers to the attribute of two quantities, that an equation has two sides, and its use as a relational symbol (Rittle-Johnson & Alibali, 1999). Participants' perception involving "determine the numbers, perform the operation, and finish" was observed commonly in proportion. However, proportional reasoning is necessary in the solution of the given problems. This form of reasoning is qualitative as well as quantitative (Lesh et al., 1988). It is stated that focusing on finding the result-the unknown by merely performing operations will not contribute to the development of students' proportional reasoning skills, on the contrary, it will lead to a focus on only operations and producing results, therefore a rule-based approach to mathematics. It is stated that this is important in terms of the acquisition of many concepts such as "congruence, relative magnitude and size, expansion, scaling, pi, constant change rate, slope, speed, different unit proportions, percentage, trigonometric proportions, probability, density" (Heinz et al., 2008, p. 508). Logical conjunctions help to make connections between the various steps of mathematical actions in terms of meaning and form. The order in operations or mathematical steps is established by means of conjunctions. Therefore, mathematical conjunctions provide integrity in terms of content and expression in mathematical arguments or texts. The errors and ambiguity in participants' proportion representations, operations with rational numbers and particularly the presentation of solutions may be attributed to the fact that logical conjunctions were used incorrectly and inefficiently, or not used at all. It can be said that one of the main reasons why the models and symbolic representations produced by participants in rational numbers were coalesced into one and presented in an intermingled way in terms of the operation order is that they used logical conjunctions inefficiently. Similarly, one of the main reasons behind the poor comprehensibility of the solutions may be that participants did not possess adequate knowledge regarding the logical language. With the aim of increasing the awareness of pre-school teachers regarding the potential future language barriers in learning mathematics, Keuch and Brandt (2018) revealed certain linguistic structures that may be harmful in terms of the concept of length. It can be said that primary prospective mathematics teachers need similar awareness. As aforesaid, the mathematical texts written by the prospective teachers were difficult to follow and comprehend, and that participants followed a series of steps involving "determine the numbers, perform the operation, and finish". The underlying reasons for these solutions may be that participants focused only on the result, did not place enough importance on the conceptual basis of the solution. They focused on determining and applying operations by overlooking the comprehension of the solutions and the means of expression and adopting a result-oriented approach. It can be said that this approach is not favorable for a prospective teacher. The challenge to solve as many problems within the shortest time period as part of the Turkish education system, which was designed with large-scale central exams (Yıldırım, 2008), may be among the reasons behind this tendency. Despite this, prospective teachers are required to have developed their mathematical language skills throughout their undergraduate education. However, this is also attributed to prospective teachers' sense of professional liability.

Conclusion, Implications and Limitations

Considering that writing can provide linguistic understandings beyond the speaking (Whitin & Whitin, 2000) and that the contributions of integrating writing in mathematics teaching (Adu-Gyamfi et al., 2010; Baxter, 2008; Bicer et al., 2013; Ntenza, 2006; Pugalee, 2001), the present study may be insightful in terms of prospective teachers' competence of using mathematical language. In the present study, it was observed that there were incomplete and incorrect uses of language in the mathematical texts written by the prospective teachers to a degree that would impact their mathematical communication with students. While the importance of linguistic competence in learning and performing mathematics is known (Muzvehe & Capraro, 2012; Kim et al., 2012), the type of linguistic competence adequate for learning and performing mathematics is discussed in the literature (Morgan et al., 2005), and this is a subject of curiosity for teacher-training institutions as well. In this context, it can be said that the findings and the categories related to the use of language determined in the present study also clarified the linguistic factors to be taken into consideration in terms of learning and performing mathematics.

In this study, prospective teachers' competence of using mathematical language in mathematical texts were examined in the context of problem solving. In this context, the diversity of symbols, conjunctions and terms used by teachers is limited to the content of the problems presented to them, as well as prospective teachers' competence of using mathematical language. Therefore, this competence can also be examined by asking prospective teachers to prepare mathematical texts in different contexts, such as proof, writing for the purpose of teaching mathematics, through future scientific studies. However, it should be noted that mathematical language is definite by its symbols, conjunctions, and usage. It is important that this language is used correctly and completely whenever mathematics is done. It is not desirable for the mathematics teachers, to use the mathematical language carelessly and incorrectly in the classrooms, just to be practical. Incorrect and incomplete use of language is acceptable for a primary school student, provided that it is corrected and improved, but not for a teacher or prospective teacher.

In terms of the mission they have acquired and their future roles, prospective teachers are expected to write mathematically complete and correct mathematical texts in all contexts and conditions.

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