Problem Solving Strategies of Selected Pre-service Secondary School Mathematics Teachers in Malaysia

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ABSTRACT

Problem solving strategies of eight pre-service secondary school mathematics teachers (PSSMTs) were examined in this study. A case study research design was employed and clinical interview technique was used to collect the data. Materials collected for analysis consisted of audiotapes and videotapes of clinical interviews, subjects' notes and drawings, and researchers' notes during the interview. Strategies used by PSSMTs to solve a problem called "the fencing problem" were identified. Findings of the study suggest that the subjects used combinations of different strategies to solve problems. The implications of the finding were also discussed.

Keywords: *Problem solving strategies, Case study, Clinical interview.*

INTRODUCTION

Problem solving is as an important life skill in the 21st century and an integral component of teacher education programs in Malaysia. Trainee teachers were made to solve problems regularly to help them strengthen their content and pedagogical knowledge. Various strategies can be used to solve problems. Among the strategies recommended by the Ministry of Education Malaysia (2003) are as follows: *trying a simple case; trial-and-error (also known as guess-and-check); drawing diagrams; identifying patterns; making a table, chart, or systematic list; simulation; using analogies; working backward; logical reasoning; and using algebra* (p. 4). What strategies do pre-service secondary school mathematics teachers (PSSMTs) employ to solve problems and check their results? The present paper attempted to answer this question.

Theoretical framework

The theoretical framework for this study is based on Polya's four problem solving stages; *understand the problem, devise a plan, carry out the plan and look back or check the answer*. *Looking back* provides the opportunity for students to examine the solution through activities such as checking and verifying the result, checking the argument, deriving the result differently, using the result or the method for some other problem, reinterpreting the problem, interpreting the results or stating a new problem to solve (Polya, 1973). Despite being a very important part of problem solving, it is often neglected by teachers because of the entrenched belief that problem solving is more to get an answer above others (Wilson, 1990). Research reports have long ago shown that *looking back* is hard to accomplish and found little evidence among students of looking back (Kantowski, 1977).

It is interesting to note that, in Malaysia, the constructivist theories have received considerable acceptance in mathematics education since the late 1990s. Constructivism is consistent with current cognitive theories of problem solving and mathematical views of problem solving involving exploration, pattern finding, and mathematical thinking (Kaput, 1977; Lochhead, 1977; Schoenfeld, 1988). Thus teachers and teacher educators were urged to become familiar with constructivist views and evaluate these views for planning their approaches to teaching, learning, and research dealing with problem solving. In the constructivist perspective, learners must be actively involved in constructing their own knowledge rather than passively receiving knowledge. The teacher's responsibility is to arrange situations and contexts within which the learners construct appropriate knowledge (Glasersfeld, 1991).

Aims and research questions

The aim of this study was to examine the problem solving strategies used by eight pre-service secondary school mathematics teachers (PSSMTs) and how successful they were in solving the problem and justifying their answers. Specifically the focus was only on one of the problem solving tasks used in Wun's doctoral thesis (2010) namely the "fencing problem". The research questions addressed were:

(a) What are the strategies used by pre-service teachers to solve the 'fencing problem'?
(b) What are the strategies used by pre-service teachers to check the correctness of their answers?

METHODS

A case study research design was used to examine problem solving strategies of PSSMTs. Purposeful sampling was employed to select the eight subjects (sample) for this study. They were PSSMTs from a public university in Peninsular Malaysia enrolled in a 4-year Bachelor of Science with Education (B.Sc.Ed.) program, majored or minored in mathematics. These subjects enrolled for a one-semester mathematics teaching methods course during the data collection of this study. The mathematics teaching methods course was offered to B.Sc.Ed. program students who intended to major or minor in mathematics. The researchers had selected four B.Sc.Ed. program students who majored in mathematics, and four B.Sc.Ed. program students who minored in mathematics for the purpose of this study. All of them do not have any teaching experience prior to this study. Each PSSMT was given a pseudonym, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, in order to protect the anonymity of all interviewees. The brief background information about the subjects is shown in Table 1.

Table 1 Subjects' Gender, Age, Major, Minor, and CGPA

Subject	Gender	Age	Major	Minor	CGPA
Usha	Female	(21, 9)	Mathematics	Biology	2.92
Mazlan	Male	(21, 8)	Mathematics	Chemistry	2.70
Patrick	Male	(21, 7)	Mathematics	Chemistry	3.04
Beng	Female	(22, 9)	Mathematics	Physics	3.82
Roslina	Female	(21, 8)	Biology	Mathematics	3.15
Liana	Female	(21, 5)	Chemistry	Mathematics	2.77
Tan	Male	(22, 7)	Chemistry	Mathematics	3.69
Suhana	Female	(20, 10)	Physics	Mathematics	2.52

Instrumentation The interview task was adapted from Sgroi (2001, p. 181).

A gardener has 84 m of fencing to enclose a garden along three sides, with the fourth side of the garden being formed by a wall. (Assume that the wall is perfectly straight). What are the dimensions of a rectangular garden that will yield the largest area being enclosed?

During the individual clinical interview, the task which was written on paper was shown to the participant. They were told that suppose one of their students asks for help with the problem, how would they solve the problem. Besides determining the strategies used to solve the problems, strategies used by the subjects to check the correctness of their answers were also determined.

Data collection and analysis procedures

Data for this study were collected using one-on-one clinical interview techniques. Each interview was recorded. Materials collected for analysis consisted of audiotapes and videotapes of clinical interview, subject's notes and drawings, and researcher's notes during the interview.

The data analysis process comprised four levels. At level one, the audio and video recording of the clinical interview were verbatim transcribed into written form. The transcription included the interaction between the researcher and the subject during the interviews as well as the subject's nonverbal behaviors. At level two, raw data in the forms of transcription were coded, categorized, and analyzed according to specific themes to produce protocol related to the description of the problem solving strategies of PSSMTs. At level three, case study for each subject was constructed based on information from the written protocol. At this level, analysis was carried out to describe each subject's behaviors in solving the task. At level four, cross-case analysis was conducted. The analysis aimed at identifying patterns of responses of problem solving by the subjects. Based on this pattern of responses, problem solving strategies of PSSMTs were summarized.

FINDINGS

Table 2 summarized the findings. Four of the subjects successfully solved the problem namely Beng, Patrick, Suhana, and Tan while another four namely Liana, Mazlan, Roslina, and Usha, have made attempts but were unsuccessful in solving the fencing problem. Most of the PSSMTs used a combination of strategies to solve the fencing problems.

Of the four PSSMTs who have successfully solved the fencing problem, Beng and Patrick used a combination of *draw a diagram, making a list, trial and error* and *identifying patterns* to solve the fencing problem. Suhana meanwhile used a combination of *draw a diagram* and *systematic trial-and-error strategy* successfully while Tan used a combination of *draw a diagram*, *equation and differentiation* method to solve the fencing problem. Of the four PSSMTs who were unsuccessful in solving the fencing problem, two of them, namely Mazlan and Roslina had attempted to use the *draw a diagram* and *trial-and-error* strategy. Usha tried to use the *draw a diagram* and *identify a pattern* strategy; Liana chose the *differentiation method* to solve the fencing problem but was not successful. Table 6 reveals the strategies used by PSSMTs to solve the fencing problem.

In addition, the subjects were also asked to justify whether their answers were correct. Three of the PSSMTs, namely Beng, Patrick, and Usha, used the *identifying pattern* strategy to check the answer for the fencing problem, Suhana used the *compare strategy* to verify the answer and Tan checked the answer of the fencing problem by calculating the value of $\frac{d^2A}{dx^2}$ at the stationary point. Mazlan used the same strategy, namely *trial and error* strategy, to verify the answer and Roslina suggested that she would use the *list all-and-compare strategy*, to verify the answer.

Table 2Summary of Strategies Used for Solving the Problem and Verifying the Answer

Subjects	Successful in solving?	Strategies used to solve problem	Strategies used to verify
Beng	Yes	Draw a diagram, making a list, unsystematic Trial and error and identifying pattern	Identifying pattern
Patrick	Yes	Draw a diagram, making a systematic listing and Identifying pattern	Identifying pattern
Usha	No	Draw a diagram, Identifying pattern	Identifying pattern
Mazlan	No	Draw a diagram, Trial and error	Trial and error
Suhana	Yes	Draw a diagram, Systematic Trial and error	Compare
Roslina	No	Draw a diagram, Trial and error	List-all-and compare
Tan	Yes	Draw a diagram, equation and Differentiation	Double differentiation
Liana	No	Draw a diagram, Differentiation	Unable to verify

Strategies employed by PSSMTs to solve the Fencing problem.

A. Combination of draw a diagram, making a list, trial and error and identifying pattern

Beng solved the problem successfully using a combination of *draw a diagram*, *trial and error* and *identifying pattern* strategy. He provided an accurate explanation of the procedures. Excerpt 1 is illustrative of the strategies. R refers to the researcher while S refers to subject.

Excerpt 1

R: …How would you solve this problem?
S: (Draws the following diagram).

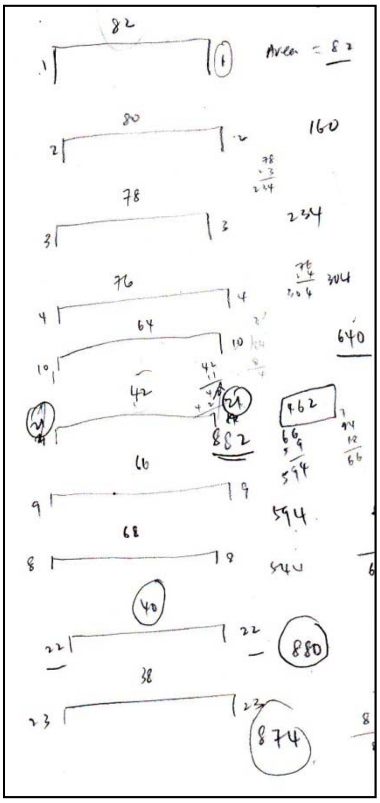

Figure 1. Beng uses looking for a pattern strategy to solve the fencing problem.

R: Could you explain your solution?

S: I'll ask them to try first. If here is 1, 1, here is 82 and the area is 82 and will be increasing. So, I just ask them to take the half of the value. 84, half is 42 and this one will be 21 and 21. So, the value they get will be 882. Then I need to test again. Ask them to use, increase this number; 22, 22, this one 40 and the value [they] get is 880. Continue with 23, 23, 38. The value is decreasing. So, the trend is increasing up to this point and then decreasing. So, the dimension for the largest area will be 42 times 21.

In Excerpt 1, Beng started off with the width and the length of the rectangular garden as 1 m and 82 m respectively and this yielded the smallest area being enclosed, namely 82 m^2. She then increased the width of the rectangular garden, one meter at a time, to 4 m and reduced the length of the rectangular garden accordingly to 76 m. Consequently, the area increased to 304 m^2. Beng saw a pattern that area increases as she increases the width of the rectangular garden while reducing its length accordingly. She increased the width of the rectangular garden to 10 m instead of 5m and reduced its length to 64 m. The area increased to 640 m^2. Subsequently, Beng took half of the 84 m of fencing as length of the rectangular garden and 21 m as its width. The area now increased to 882 m^2.

Beng attempted to verify whether 882 m^2 was the largest area being enclosed. She tested it with two values of the width that were smaller than 21 m, namely 9m and 8 m respectively. Beng found that the area decreased to 594 m^2 and 544 m^2 respectively. Beng also tested it with two values of the width that were larger than 21 m, namely 22 m and 23 m respectively. Beng found that the area decreased to 880 m^2 and 874 m^2 respectively. Thus, Beng concluded that 882 m^2 is the largest area being enclosed and the dimension of the rectangular garden that yields the largest area being enclosed is 42 m by 21 m. Table 3 summarizes the dimensions of the rectangular garden and its area that Beng has figured out.

How Beng Checked the correctness of her answer

When probed to check and verify the dimension of the rectangular garden that yields the largest area being enclosed, Beng reflected on the solutions that she has figured out, as shown in Figure 1. Beng explained that she started off with the smallest value of the width of the rectangular garden, namely 1 m, with the length of the garden as 82 m. Beng explained that its area increased to 882 m^2 as she increased the width to 21 m and reduced the length of the rectangular garden to 42 m. Beng realized that its area kept decreasing after that when she increased the width of the rectangular garden to 22 m and 23 m respectively and reduced its length accordingly. Thus, Beng reiterated that the maximum area, 882 m^2, occurred at the “center” when its dimension is 42 m by 21 m. Excerpt 2 is illustrative.

Excerpt 2

R: How do you know that the dimensions will give you the largest area?

S: Mm ... I just use sequence because I test from the largest one here (points to diagram with the value of 21, 42, 21, as shown in Figure 1). Start here with the smallest one (points to diagram with the value of 1, 82, 1, as shown in Figure 1) and it keeps increasing. (Draws the fence with the shortest length (2 m) and the longest width (41 m)). I think when it goes back, dwell down, finally here will become two and the value is smaller and smaller. So, the maximum value will be at the center. That's why I try the center.

Table 3 Dimensions of Rectangular Garden and its Area by Beng

Length (m)	Width (m)	Width (m)	Area (m^2)
82	1	1	82
80	2	2	160
78	3	3	234
76	4	4	304
64	10	10	640
42	21	21	882
66	9	9	594
68	8	8	544
40	22	22	880
38	23	23	874

B. Combination of using diagram, equation and differentiation method

Tan used diagram, equation and differentiation method to solve the fencing problem. Excerpt 3 is illustrative of Tan’s strategies.

Excerpt 3

R: …How would you solve this problem?

S: (Draws the following diagram to represents the fencing of the rectangular garden)

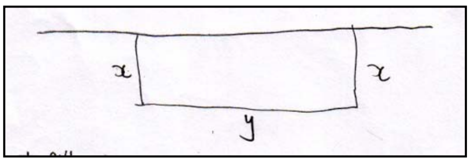

Figure 2. Tan draws a diagram to represents the fencing of the rectangular garden.

(He then writes down the following)

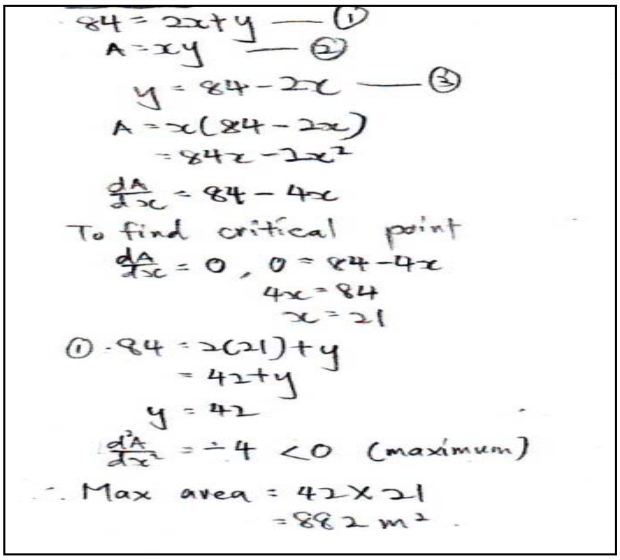

Figure 3. Tan uses the differentiation method to solve the fencing problem.

R: Could you explain your solution?

S: Okay. I'll explain from the beginning. Given that the gardener has 84 m of fencing. So, to get a rectangle, given that the fence to a wall, I come out with a formula, $84 = 2x + y$ (Refer to Figure 2 and Figure 3). So, this is the perimeter that the fence will take, the general formula. So, the area is since the two sides of it must be the same, so x times y, I can get the area. So, there are two equations. But that's a problem, got two variables. I need to eliminate one of the variables in order to perform differentiation. So, what I do is I substitute y to eliminate it and then left out x only. So, with area we get function of x only, I can perform the differentiation. So, what I get after differentiate it is $\frac{dA}{dx} = 84 - 4x$. To find the critical point, the $\frac{dA}{dx}$ must be assigned to zero. After I assigned the zero, I get x is 21. After that I substitute this as 21 to the perimeter equation. So, I get the final y is 42. But before I find the largest area, I need to prove that this differentiation is maximum. So, I differentiate twice, I get minus 4 (refers to -4). So, it's smaller than zero. So, it can be deduced that it is a maximum.

In Excerpt 3, Tan drew a diagram to represent the fencing of the rectangular garden. He then used the equation and differentiation method to solve the fencing problem. Tan wrote equation ① to represent the perimeter of the fencing. He wrote equation ② to represent the area of the rectangular garden. Tan explained that he needed to eliminate one of the variables, namely y, in order to find the derivative ($\frac{dA}{dx}$). Thus, Tan rewrote the equation ① as $y = 84 - 2x$ and labelled it as equation ③. He substituted $y = 84 - 2x$ into equation ② and simplified it as $A = 84x - 2x^2$. After differentiating with respect to x, Tan got the derivative $\frac{dA}{dx} = 84 - 4x$. At the stationary point, $\frac{dA}{dx} = 0$ and he got $x = 21$. Tan substituted the value of x into equation ① and got $y = 42$.

How Tan checked the correctness of his answer

Tan justify his solution by differentiating twice.

Why is it a maximum? Because when you differentiate twice and you get a negative value, which is smaller than zero, then straight away the equation can be said that it's … a maximum equation. So, after confirming that it is a maximum, straight away I can find the maximum area by times the two variables, 42 times 21 and I get the final answer 882.

Tan elaborated that he needed to find the value of $\frac{d^2A}{dx^2}$ at the stationary point. If $\frac{d^2A}{dx^2} < 0$, then the point is at a maximum. Tan found that $\frac{d^2A}{dx^2} = -4 < 0$ and thus (21, 42) is a maximum point. Tan concluded that 882 m^2 was the largest area being enclosed and 42 (m) by 21 (m) was the dimension of the rectangular garden that will yield the largest area being enclosed.

C. Combination of using draw a diagram, making a systematic list and identifying pattern

Patrick used *draw a diagram*, *making a systematic list* and *identifying pattern* strategy to solve the fencing problem. Excerpt 4 is illustrative of his strategies.

Excerpt 4

R: …How would you solve this problem?

S: (Draws the possible rectangular gardens and calculates their respective areas)

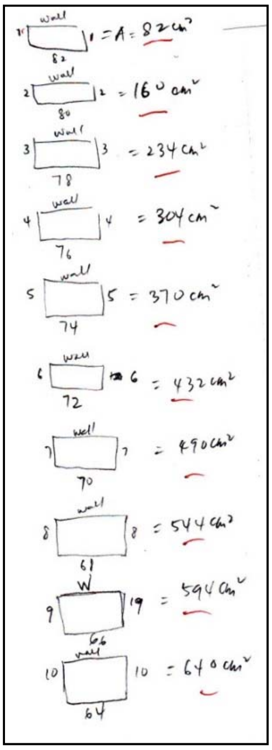

Figure 4 Patrick draws the possible rectangular gardens and calculates their areas.

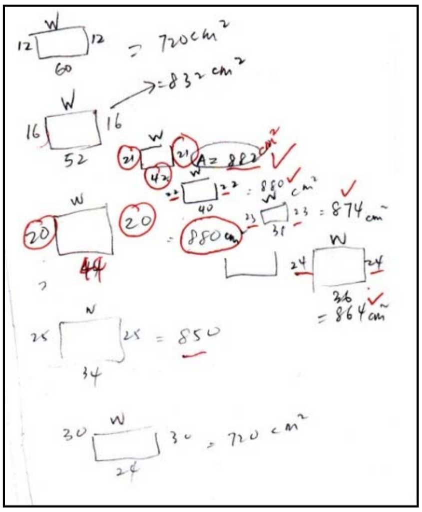

Figure 5. Patrick continues to draw the possible rectangular gardens and calculates their areas.

R: Could you explain your solution?

S: Okay, here we can find out by looking for a pattern. So, here if we put 1, 1, 82, the area keeps on increase. But then until, it reaches a level where the opposite sides are 20 and the remaining side is 44. This is 880 square meter and here we want to make it step by step: 21, 22, 23. 21, 882. When this side (point to the width of the rectangle) becomes 22, 880. 23, 23, 874. So, 24, it will (be) 864. 25, it will increasingly decrease. So, the peak is when this side is 21 (point to the width of the rectangle), and here 42 (point to the length of the rectangle).

R: So, what are the dimensions that will give you the largest area?

S: 21, 42.

R: What is your maximum area?

S: 882.

In Excerpt 4, Patrick started off by drawing a diagram and making a list with the width and the length of the rectangular garden as 1 m and 82 m respectively and this yielded the smallest area being enclosed, namely 82 m^2. He then increased the width of the rectangular garden, one meter at a time, to 10 m and reduced the length of the rectangular garden accordingly to 64 m. Consequently, the area increased to 640 m^2, as shown in Figure 4. Patrick saw a pattern that area increases as he increases the width of the rectangular garden while reducing its length accordingly. He increased the width of the rectangular garden to 12 m instead of 11 m and reduced its length to 60 m. The area increased to 720 m^2, as shown in Figure 5.

Subsequently, Patrick increased the width of the rectangular garden, four meters at a time, from 12, to 16 and then 20 m and reduced the length of the rectangular garden accordingly from 60, 52 and to 44 m. He then increased the width from 21, 22, 23 and 24. Consequently, the area increased to 882m^2, and then went down to 880 m^2 and 874m^2 respectively. He then increased the width of the rectangular garden, five meters at a time, to 30 m and reduced the length of the rectangular garden accordingly to 24 m. Consequently, the area decreased to 720 m^2. Patrick realized that the area of the rectangular garden decreased when he increased the width of the rectangular garden, five meters at a time, from 20 m to 25 m and reduced the length of the rectangular garden accordingly from 44 m to 34 m. Consequently, the area decreased from 880 m^2 to 850 m^2.

Patrick became more cautious and he decided to increase the width of the rectangular garden, one meter at a time, from 20m to 21m and reduced the length of the rectangular garden accordingly from 44 m to 42 m. Consequently, the area increased from 880 m^2 to 882 m^2. Patrick continued to increase the width of the rectangular garden, one meter at a time, to 24m and reduced the length of the rectangular garden accordingly to 36m. Consequently, the area decreased to 864 m^2. Patrick concluded that 882 m^2 was the largest area being enclosed. He circled the answer and then put a "longer" tick (√) behind it, as shown in

Figure 5. Table 4 summarizes the dimensions of the rectangular garden and its area that Patrick has figured out.

Table 4 Dimensions of Rectangular Garden and its Area as Patrick Calculated

Length (m)	Width (m)	Width (m)	Area (m^2)
82	1	1	82
80	2	2	160
78	3	3	234
76	4	4	304
74	5	5	370
72	6	6	432
70	7	7	490
68	8	8	544
66	9	9	594
64	10	10	640
60	12	12	720
52	16	16	832
44	20	20	880
34	25	25	850
24	30	30	720
42	21	21	882
40	22	22	880
38	23	23	874
36	24	24	864

How Patrick confirmed the correctness of his answer

To check his solutions, Patrick used the identifying pattern method. Patrick expressed that the area kept on increasing as he increased the width of the rectangular garden and reduced the length of the rectangular garden accordingly. Patrick noticed that when he increased the width of the rectangular garden to 20 m and reduced its length to 44 m, the area increased to 880 m^2. Patrick found that when he further increased the width of the rectangular garden, one meter at a time, from 20 m to 25 m and reduced the length of the rectangular garden accordingly from 44 m to 34 m, he noticed that the area increased from 880 m^2 to 882 m^2, and then kept on decreasing from 882 m^2 to 850 m^2. Thus, Patrick pointed out that it reached its "peak" (the largest area being enclosed) when the width and the length of the rectangular garden is 21 m and 42 m respectively. Patrick reiterated that 882 m^2 was the largest area being enclosed and 42 m by 21 m is the dimension of the rectangular garden that will yield the largest area being enclosed.

D. Combination of draw a diagram, Systematic and guided trial and error method based on factor

Suhana used a combination of *draw a diagram* and *systematic guided trial and error* method to solve the fencing problem. Excerpt 6 is illustrative of her strategies.

Excerpt 6

R: …How would you solve this problem?

S: (List down the possible factors of 84. There is an error in the second row. The actual factors in the second row should be 2 and 42, not 41, as shown in Figure 6 and then she proceeds to use the factors to solve the problem)

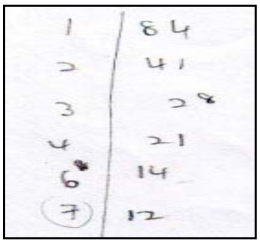

Figure 6 Suhana first listed down all the possible factors of 84.

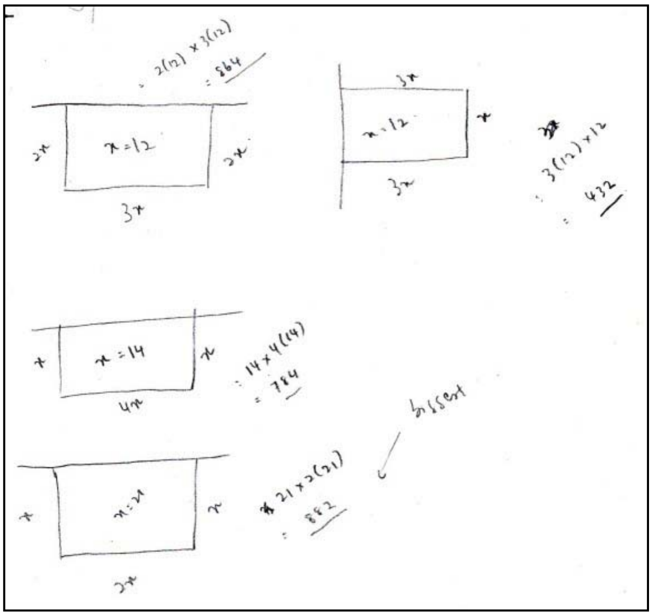

Figure 7. Based on the factor of 84 Suhana used trial and error method tosolve the fencing problem.

R: Could you explain your solution?

S: Okay. I ask them to draw the rectangle (points to the rectangle with the dimensions of 2*x* and 3*x*, as shown in Figure 7).

R: What is your "*x*"?

S: "*x*" equal to 12. So, $2x \times 3x = 24 \times 36 = 864$ (uses calculator to get the product). And then second (points to the rectangle with the dimensions of *x* and 4*x*, as shown in Figure 7), $x \times 4x = 14 \times 56 = 784$ (uses calculator to get the product. Third, (points to the rectangle with the dimensions of *x* and 2*x*), $x \times 2x = 21 \times 42 = 882$ (uses calculator to get the product). And then fourth, (points to the rectangle with the dimensions of 3*x* and *x*), $3x \times x = 36 \times 12 = 432$ (uses calculator to get the product). (Draws an arrow pointed towards 882 and writes the word "biggest" to indicate "882" is the largest area being enclosed, as shown in Figure 7).

In Excerpt 6, Suhana drew a diagram to list down the possible factors of 84. There was an error in the second row. The actual factors in the second row should be 2 and 42, not 41, as shown in Figure 7. Based on the list of factors of 84, she used the trial and error method to solve the fencing problem by identifying the factors that yield the largest area, as shown in Figure 8. In the first trial, Suhana viewed 7 as the sum of 2, 3, and 2, and drew a rectangle with the dimension of 2*x* by 3*x*. She calculated the area of the rectangle as $2x \times 3x = 24 \times 36 = 864$, where $x = 12$.

In the second trial, Suhana viewed 6 as the sum of 1, 4, and 1, and drew a rectangle with the dimension of *x* by 4*x*. She calculated the area of the rectangle as $x \times 4x = 14 \times 56 = 784$, where $x = 14$. In the third trial, Suhana viewed 4 as the sum of 1, 2, and 1, and drew a rectangle with the dimension of *x* by 2*x*. She calculated the area of the rectangle as $x \times 2x = 21 \times 42 = 882$, where $x = 21$. In the fourth trial, Suhana viewed 7 as the sum of 3, 1, and 3, and drew a rectangle with the dimension of 3*x* by *x*. She calculated the area of the rectangle as $3x \times x = 36 \times 12 = 432$, where $x = 12$. Suhana drew an arrow pointed toward 882 and wrote the word "biggest" to indicate "882" is the largest area being enclosed, as shown in Figure 7.

How Suhana checked the correctness of her answer

In Excerpt 7, Suhana compared the areas of the rectangular garden that she had calculated. Suhana indicated that 882 was the largest area among the areas that she had calculated, namely 864, 784, 882, and 432. Thus, Suhana concluded that 882 (m^2) is the largest area being enclosed. She stated that 42 (m) by 21 (m) is the dimension of the rectangular garden that will yield the largest area being enclosed.

Excerpt 7

R: How do you know "882" is the largest area?
S: By comparing these four figures (points to 864, 784, 882, and 432 in Figure 7).
R: What are the dimensions that give you the largest area?
S: 21, 42, and 21

Strategies employed by subjects who were unsuccessful in solving the problem

The other four subjects who were unsuccessful in solving the problems attempted to solve it using several strategies but failed to get the correct answer. Usha for example was not systematic in listing the length and width of the rectangle and in the process missed the correct answer. Liana seemed to have limited knowledge of solving the problem as shown in the following excerpts:

…Well, the formula for the area is "a times b" and then perimeter is "a plus 2b". This is for the fence because the fence is required here only. So, I didn't add up the wall here. So, "a plus 2b" actually is same amount of 84. So, I required another equation to solve these two unknowns. The problem that I faced here is how to manipulate the equation of the area here. Okay as I recall that's something have to do with the two equations here. But I just can't remember what it is.

Mazlan seemed to have problems with understanding the problem and representations in diagram form. For example he did not understand that the fourth side is a wall (refer to Figure 8). He also seemed to have problems with the concept of perimeter.

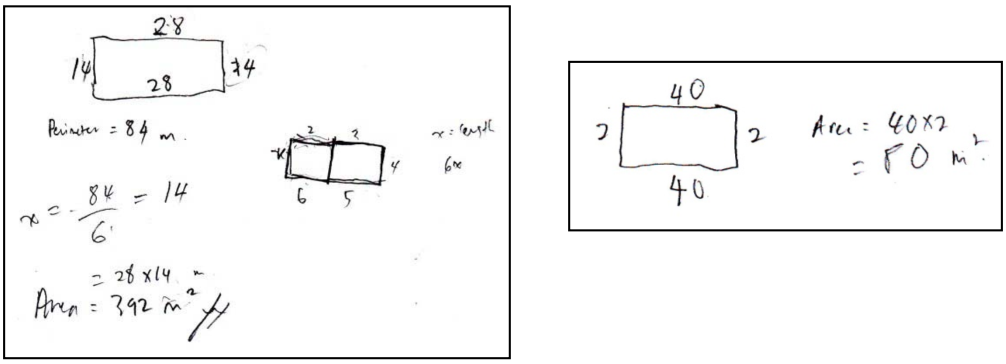

Figure 8 Mazlan's representation of the problem.

Roslina on the other hand had misconceptions that longer length means bigger area:

So, to get the largest area we enclosed, I assume that this is 80 m, this is 2m and this is 2m. So, this is 80, 82, 84 m fence. Then to get the area, 80 times 2, I get 160 m^2 (misreads 160 m^2 as 160 m square)

..My first choice is 80 and 2. So, 84 and then my other choice would be for example 82 and 1 (shown in Figure 10)… If I want to get the area of the rectangle, this will be 82 times 1. So, this is 82 m^2 (misreads 82

m^2 as 82 m square). So, surely that this is the largest area being enclosed (refers to her first choice, as shown in Figure 9). So, I think I'll say that this is the largest area.

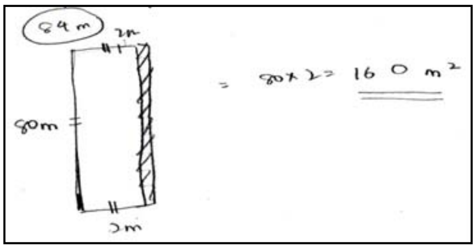

Figure 9

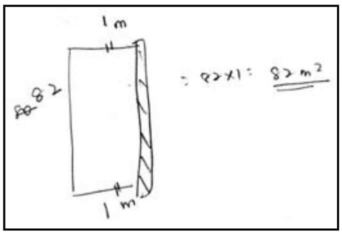

Figure 10

DISCUSSION AND CONCLUSIONS

This study reported on the strategies employed by eight pre-service secondary school mathematics teachers (PSSMTs) to solve a problem namely the "fencing problem" which focused on the concept of perimeter and area. Four of the eight were able to solve the problem using a combination of strategies but the other four were unable to solve the problem despite attempts made. The conclusions from this study are as follows:

Most of the pre-service teachers used not one specific strategy but a combination of strategies to solve the problem. The most common strategy is to draw a diagram followed by other strategies such as trial and error, identifying pattern, using equation or listing.

Pre-service teachers were found to use different levels of strategies. The strategies used by the PSSMTs ranged from the primitive unsystematic trial and error to a more advanced strategy using prior knowledge of factors, equations and differentiation.

Pre-service teachers who were unable to solve the problem correctly seemed to use limited and incorrect mathematical terminology, lack understanding of the problem, were unable to make representations of the word problems, lack some basic knowledge, and had misconceptions regarding length of a rectangle and its area.

These findings are quite disturbing; misconceptions are still prevalent among the pre- service teachers who either majored or minored in mathematics. Lacking mathematical terminology is also of concern as mathematics teachers should be able to communicate clearly and accurately when teaching. The seemingly obvious misconceptions could be an indication of inadequate knowledge expected among the pre-service teachers to teach area and perimeter in future. As teacher educators, we need to be mindful of this and provide opportunities for pre-service teachers to address the gaps in their problem solving knowledge. Perhaps what is needed is to provide PSSMTs with more opportunities to examine student misconceptions and engage in diagnostic activities.

Most of the pre-service teachers had limited ideas of "looking back" as merely checking the results. The process of looking back was not properly conducted by the pre-service teachers in this study. Certain subjects were merely repeating the steps taken to solve the problem instead of checking and verifying whether the answer was correct. Other activities of looking back as suggested by Polya (1973) were not used by them. This finding is consistent with those noted by Kantowski (1977) and Wilson (1990).

This study only involved eight PSSMTs enrolled in the 4-year Bachelor of Science with Education (B.Sc.Ed.) program in a public university in Peninsular Malaysia. Thus, the findings of this study could not be generalized to other PSSMTs enrolled in the 4-year Bachelor of Science with Education (B.Sc.Ed.) program in this public university, in other programs (e.g., Bachelor of Education (B. Ed.), Diploma in Education (Dip.Ed.)), or attending other universities and teacher training institutes. However, the findings could give some indication and an overview regarding the strategies pre-service teachers used in problem solving.

This study has several implications. First, pre-service teachers need to be trained to solve problems using several strategies. Some common strategies or heuristics are used in secondary schools to solve problems. But other alternative strategies can also be used. Research derived strategies can be shared with the pre-service teachers to enrich their knowledge and to narrow the knowledge gap.

Second, the looking-back stage in the Polya steps of problem solving should be given more emphasis in problem solving rather than being used merely to check answers. Other opportunities for checking the argument, deriving the result differently, using the result or the method for some other problem, reinterpreting the problem, interpreting the results or stating a new problem to solve should be exposed to pre-service teachers.

Finally, pre-service teachers should also be given opportunity to solve different types of problems. Mathematics teacher educators need to organize teaching and learning activities that provide such opportunity. Through such activities, pre-service mathematics teachers would be provided opportunity to develop their problem solving skills. This is in line with the goal of the secondary school mathematics curriculum not only in Malaysia but elsewhere in the region, namely to develop individuals who are able to think mathematically and can apply mathematical knowledge effectively and responsibly in problem solving and decision making (Ministry of Education Malaysia, 2003).

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