MALAYSIAN JOURNAL OF SCIENCE
MJS
ISSN 1394-3065

Regular Issue

Malaysian Journal of Science 44 (1): 101-120 (March 2025)
https://mjs.um.edu.my

A Review on Modern Lunar Crescent Visibility Criterion

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Abstract: The modern lunar crescent visibility criterion is a criterion produced in the 20th century, which began with Fotheringham's criterion in 1910 and continued until the present date. A limited number of research studies have been conducted on the modern lunar crescent visibility criterion, with a majority of the studies placing focus on the design and social aspects of it. Therefore, this article aims to provide a review on the modern lunar crescent visibility criterion, which covers the scientific outlook of a lunar crescent visibility criterion: data locality, prediction strength and weaknesses, and its long-term legacy in visibility. The said review is conducted using systematic literature analysis, and specific inclusion and exclusion criteria, performed based on 27 works on the lunar crescent visibility criterion that pass the selection criteria. The review suggests that a new outlook on modern lunar crescent visibility criterion study can be obtained, by conducting an assessment of collected database of lunar crescent sightings, and by providing a comparative analysis tool for modern lunar crescent visibility criterion.

***Keywords:** : Review, lunar crescent, visibility, criterion.*

1. Introduction

The criterion used to predict the visibility of a lunar crescent during an observation is called the lunar crescent visibility criterion. Researchers have developed a lunar crescent visibility criterion based on records of lunar crescent sighting and their subjective definition of lunar crescent visibility, which can be telescopic definition, naked eye definition, or above the horizon definition. A telescopic definition of lunar crescent visibility criterion is based on visible lunar crescent sighting using a telescope, while the naked eye definition means that the criterion is based on naked eye visibility of lunar crescent, while an above-the-horizon definition is a lunar crescent visibility criterion that developed based on the position of a lunar crescent above the horizon, regardless of its visibility (Faid et al., 2022).

Schaefer highlighted that lunar crescent visibility is as one of the most non-trivial research projects in the field of astronomy. This is because lunar crescent visibility is directly involved in the calendrical making of the Muslim and Hebrew calendars. Muslims require visibility of the lunar crescent to determine dates of religious importance, such as the start of the fasting month of Ramadhan, the celebration of Eid Fitri, and the period of the Muslim pilgrimage. These events require a significant amount of raw material, human resources, traffic administration, and travel planning, involving millions of dollars in transfer every year. Karaites and Samaritans, being groups of Hebrew community found in Israel, use lunar crescent sighting to determine their lunisolar calendar: The Karaites need to observe the lunar crescent to determine their Rosh Chodesh holiday, while the Samaritans have developed a computation algorithm based on data of lunar crescent observation in order to determine their calendar (Faid, Nawawi, et al., 2024; Hoffman, 2003). This demonstrates the importance of research on lunar crescent visibility criterion in the present.

Modern Lunar Crescent Visibility Criterion

Lunar crescent visibility was extensively studied during the medieval era since it was vital for purposes of calendrical making by the Islamic empire during that time (Mustapha et al., 2024). However, in keeping with the decline of science during the medieval times, the keenness to study the visibility of the lunar crescent diminished after the 16th century (King, 1991). Since then, the new Hijri month has been determined either with a lunar crescent sighting or a simple 29th-30th alternate rule. Research for lunar crescent visibility did not spark much interest among researchers until at least the 20th century (Ilyas, 1994). In 1910, Fotheringham sparked much interest in lunar crescent visibility research, followed by Maunder in 1911 and Danjon in 1936 (Danjon, 1936; Fotheringham, 1910; Maunder, 1911; Muhamad Syazwan Faid, Mohd Nawawi, and Mohd Saadon 2024). The interest then spread among the Muslim community, sparked by conflicting lunar crescent visibility reports and the determination of different dates for the new Hijri month (Moosa, 1998). This led to the first Muslim lunar crescent visibility criterion since the era of the Middle Ages, which is the Istanbul Declaration in 1976 (Mufid and Djamaluddin, 2023), followed by the Ilyas

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Received: May 28, 2023
Accepted: December 7, 2023
Published: March 31, 2025

DOI:https//doi.org/10.22452/mjs.vol44no1.12

series of lunar crescent visibility criteria, the MABIMS lunar crescent visibility criterion in 1991, and later the Fatoohi lunar crescent visibility criterion in 1998 (Fatoohi, 1998; M. Ilyas, 1984; Mohd Nawawi et al., 2015).

The modern lunar crescent visibility criteria demonstrate frequent use of altitude, azimuth, and elongation parameters, and also the introduction of width and contrast threshold to further increase the accuracy of lunar crescent visibility prediction. Modern lunar crescent visibility criteria are more composite in parameters, and its designs are constructed through a larger compilation of lunar crescent visibility reports, in contrast to the lunar crescent visibility criteria in the Middle Ages (Faid, Mohd Nawawi, Abdul Wahab, et al., 2023). Since the rekindled interest in the lunar crescent visibility criterion by Fotheringham in 1910, which we marked as a starting point for the period of modern lunar crescent visibility criterion, at least 21 lunar crescent visibility criteria have beens produced, and various social aspects of lunar crescent visibility have been explored (M.S. Faid, Mohd Nawawi, and Mohd Saadon, 2023). Among the vast available literature on the lunar crescent visibility criterion, there have been a few attempts by Ilyas, Schaefer, and Fatoohi to provide a review on lunar crescent visibility. Ilyas (1994) was among the first to review the lunar crescent visibility criteria, including the Babylonian, Hindu, and Medieval lunar crescent visibility criteria. Ilyas's review, however, was biased towards his own lunar crescent visibility parameter. The Danjon limit, for example, was interpreted based on his finding of a 10.5 degree parameter. Ilyas also did not provide any solid counterargument for contradiction as highlighted by other researchers during his time. Zainal (2001) later summarised the review by Ilyas, largely only replicating Ilyas's argument without providing any additional thought on the study of lunar crescent visibility criteria.

Schaefer (1996) published another review on lunar crescent visibility criteria, supplementing each of his arguments with 294 lunar crescent observation records, providing a solid finding for each criterion's parameter. However, he took the same path as Ilyas, although with less severity, interpreting the lunar crescent visibility criterion reviewed under the glass of his own criterion. It was also found that Schaefer had miscalculated some of his findings (Loewinger, 1995), was inconsistent in deciding his constant coefficient (Fatoohi, 1998), and was unclear on the definition of the term lunar brightness (Sultan, 2007). Fatoohi (1998) produced a comprehensive review of lunar crescent visibility criteria, using more than 500records of lunar crescent observation. Fatoohi’s review covers Ancient lunar crescent visibility criteria to Yallop's (1998) composite criteria. However, it was limited to records of lunar crescent observation that predate1998 and the criteria produced before the publication of his thesis. Fatoohi’s review was dated almost 30 years ago, and in the last 10 years, there has been no review of the modern lunar crescent visibility criterion conducted by any researcher. The limited presence of review literature on modern lunar crescent visibility criterion signifies the research gap on this matter. Therefore, this research attempts to provide a review of the modern lunar crescent visibility criterion to provide a refreshed view on the subject.

2. Methodology

To achieve the aims of our study, a review was undertaken using the PRISMA review method (Rethlefsen et al., 2021). The selection criteria for including papers are as set out in Figure 2. Selected papers were retrieved from the Google Scholar, Scopus, Mendeley, and Web of Science databases on 30 April 2023 using the search terms lunar crescent visibility criterion, new moon visibility criterion, Imkanur Rukyah criterion, and hilal visibility criterion (n = 43). Duplicate records were removed, and the study was limited to journal papers to retain robust peer-reviewed references. Then, the lunar crescent visibility criterion that was published before 1910, which is the timeframe of the modern lunar crescent visibility criterion, was excluded. Any literature published later than 1910 but contained study of criterion that predates 1910 was also excluded. The same exclusion applies for literature that is not peer-reviewed, and published in non-indexed journals. Finally, literature that discusses the social aspects of lunar crescent visibility criterion, and does not provide its lunar crescent visibility criterion was excluded as well. A total of 27 articles on the lunar crescent visibility criterion passed the exclusion criteria. The selection criteria are set out in Figure 1. Meanwhile, Table 1 portrays literature that passes the inclusion and exclusion criteria.

Literature search and selection is the first exercise conducted in this research. The literature search was conducted using Google Scholar, Scopus, Mendeley, and Web of Science databases. The search keywords are: lunar crescent visibility criterion, new moon visibility criterion, *Imkanur Rukyah,* and hilal visibility criterion. A total of 43 articles were covered using the aforementioned search and selection criteria. Then, the lunar crescent visibility criterion published before the timeframe of the modern lunar crescent visibility criterion (which is 1910) was excluded. Literature published later than 1910, but contain study of criterion that predate 1910, were also excluded. Literature that is not peer-reviewed, and published in non-indexed journals, were excluded. Finally, literature that discusses the social aspects of lunar crescent visibility criterion, and does not provide its lunar crescent visibility criterion, is excluded. A total of 27 articles on the lunar crescent visibility criterion passed the exclusion criteria. Table 1 portrays the literature that passed the inclusion and exclusion criteria.

DOI:https//doi.org/10.22452/mjs.vol44no1.12

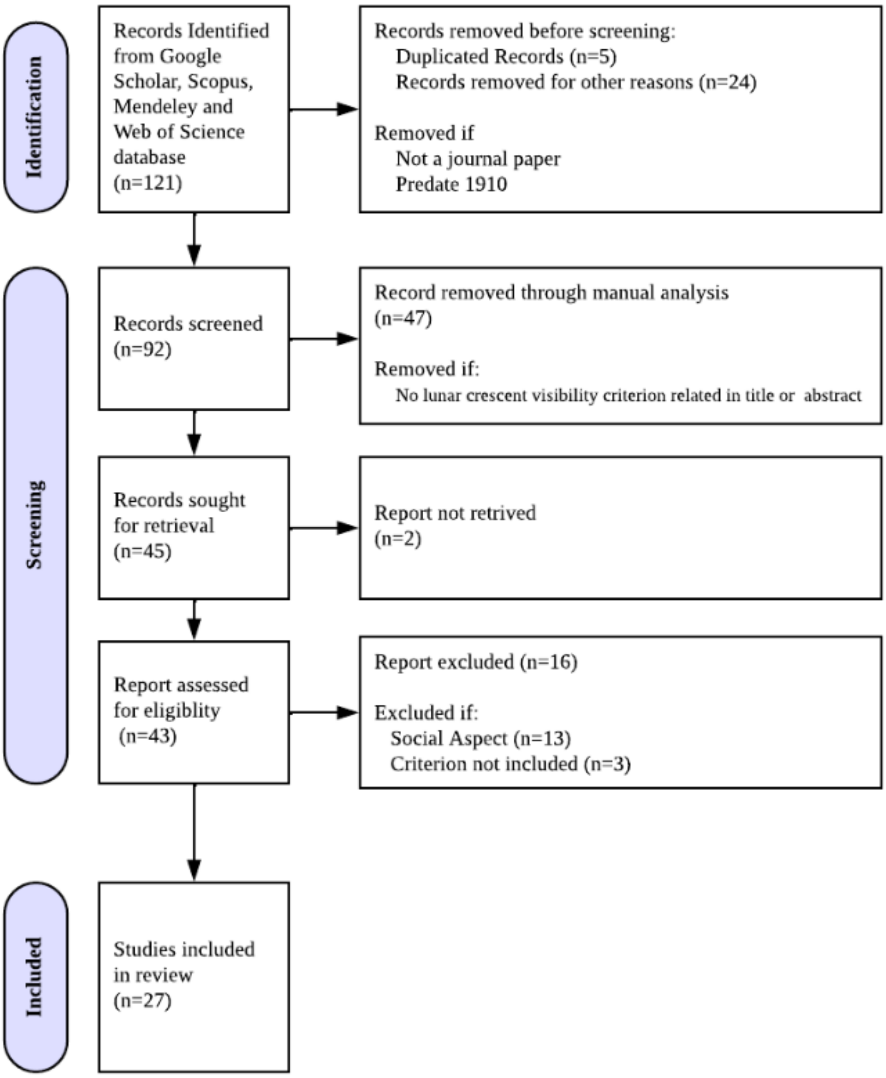


Figure 1. Selection Criteria for the literature review using the PRISMA review process.

Table 1. Lunar Crescent Criterion from Selected Publications

No	Parameter	Source	Year	Lunar Crescent Criteria Expression
1	Altitude & Azimuth	Fotheringham	1911	$ARCV \geq 12.0 - 0.008\Delta AZ$
2		Maunder	1912	$ARCV \geq 11 - 0.005\lvert\Delta AZ\rvert - 0.01\Delta AZ^2$
3		Ilyas	1984	$Arcv = -0.0027356815\, DAZ + -0.0136648716\, DAZ^2 + 0.0002119205\, DAZ^3 + 10.2832719598$
4		Fatoohi	1998	$ArcV_{Upper\ Limit} = 10.7638 + 0.0356\, \Delta Az - 0.0164\Delta Az^2 + 0.0004\Delta Az^3$ $ArcV_{Lower\ Limit} = 9.2714 - 0.0644\, \Delta Az - 0.0058\Delta Az^2 + 0.0002\Delta Az^3$
5		Krauss	2012	$Arcv^{Athenian} = 0.0291254840\, DAZ + -0.0098347831\, DAZ^2 + 0.0000475196\, DAZ^3 + 10.5981838905$
6		MABIMS 1995	1995	$ARCL \geq 3° \,\&ARCV \geq 2°$
7		Istanbul	2016	$ARCL \geq 8° \,\&ARCV \geq 5°$
8		MABIMS 2021	2021	$ARCL \geq 6.4° \,\&ARCV \geq 3°$
9	Elongation	Danjon	1936	$ARCL \geq 7°$

10		Ilyas	1983	$ARCL \geq 10.5°$
11		McNally	1984	$ARCL \geq 5°$
12		Schaefer	1991	$ARCL \geq 7.5°$
13		Fatoohi	1998	$ARCL \geq 7.5°\ for\ optical\ aided$ $ARCL \geq 9.1°\ for\ naked\ eye$
14		Odeh	2004	$ARCL \geq 6.4°\ for\ optical\ aided$ $ARCL \geq 7.7°\ for\ naked\ eye$
15		Sultan	2007	$ARCL \geq 5°$
16		Hasanzadeh	2012	$ARCL \geq 5°$
17	Singular Elongation Variable	Danjon	1936	$ARCL \geq 7°$
18		Ilyas	1985	$ARCL > 10.5$
19		McNally	1985	$ARCL > 5.0$
20	Arc of Vision & Lunar Width	Bruin	1977	$ArcV = 11.5621745317 - 7.944238328\, w' + 3.2608487770\, w'^2 - 0.4559413249\, w'^3$
21		Ilyas	1985	
22		Yallop	1998	$q = (ARCV - 11.8371 + 6.3226W' - 0.7319W'^2 + 0.1018W'^3)/10$
23		Odeh	2004	$V = ARCV - (-0.1018W^3 + 0.7319W^2 - 6.3226W + 7.1651)$
24		Qureshi	2012	$S = (ARCV - 0.351964\, W^3 + 2.222075\, W^2 - 5.422643\, W + 10.43418)$
25		Alrefay	2018	$ARCV_{Naked\ Eye} > 9.34 - 4.51w + 3.3W^2 - 1.01W^3$ $ARCV_{Optical\ Aided} > 7.83 - 4.35w + 3.22W^2 - 1.02W^3$
26	Lag Time & Elongation	Caldwell	2010	$lag\ (') > -0.9709\ arcl + 44.65\ for\ naked\ eye\ sighting$ $lag\ (') > -1.9230\ arcl + 43.13\ for\ optical\ aided$
27	Gautschy	Lagtime & Arc of Vision	2014	$Latime = 0.3342328913\, \Delta AZ + -0.0715608980\, \Delta AZ^2 + 0.0009924422 * \Delta AZ^3 + 33.8890455442$

The purpose of this review paper is to gather fragmented knowledge and place them into a single documentto provide a detailed perspective on the current state of lunar crescent visibility research. It should be noted that there are other reviews available that have a more focused method of conducting lunar crescent sighting criterion study, such as a comprehensive assessment of the lunar crescent visibility criterion (Fatoohi, 1998). Some articles consider a particular analysis viewpoint, such as lunar cycle analysis (Rahimi and Zainal, 2019; Rodzali and Man, 2021), while others focus on a specific parameter, such as the histogram bias analysis (Doggett et al., 1994), or provide only literature review without technical analysis, such as Zainal (2001). This review identifies the background and theory used to develop a lunar crescent visibility criterion by each researcher and examines the locality of the data used for the criterion developed. Each developed lunar crescent visibility criterion is portrayed in a table to enable its reconstruction using the regression technique. Each lunar crescent visibility criterion's strengths, weaknesses, and long-term impact of visibility research are highlighted to provide a neutral outlook of it. This review is critical and timely as studies on the lunar crescent visibility criterion are growing rapidly in number (Utama et al., 2023; Zulkeflee et al., 2022). The information given will aid those who are interested in the topic to gain an understanding of key methods and their applications. This review aims to highlight the solutions used in the past and identify ways in which they can be used and improved in the future.

3. Review of Modern Lunar Crescent Visibility Criterion

Studies on the visibility of lunar crescent diminished after the 16th century, in parallel with the fall of Islamic Science during the Middle Ages (King, 1993). Since then, the new Hijri month is determined either by way of lunar crescent sighting or by using the simple 29th-30th alternate rule. Research on the lunar crescent visibility limit did not garner much interest until the 20th century (M. Ilyas, 1997a). In 1910, Fotheringham sparked the interest in lunar crescent visibility limit research, followed by Maunder in 1911 and Danjon in 1936. Interest in the topic then rekindled among the Muslim community, following conflicting lunar crescent visibility reports and different dates applied for new Hijri months (Mohammad Ilyas, 1984). This led to the first Muslim lunar crescent visibility criterion since the era of the Middle Ages, Istanbul Declaration in 1976 (Rodzali & Man, 2021), followed by the Ilyas series of lunar crescent visibility criterion, the MABIMS lunar crescent visibility criterion in 1991, and the Fatoohi lunar crescent visibility criterion in 1998. The modern lunar crescent visibility criterion demonstrates frequent use of altitude, azimuth, and elongation parameter. Modern lunar crescent visibility criterion also saw the introduction of width and contrast threshold to further increase the accuracy of lunar crescent visibility prediction. Modern lunar crescent visibility criteria are more composite in parameter and their design constructed through larger compilation of lunar crescent visibility reports, in contrast to lunar crescent visibility criterion during the Middle Ages.

Fotheringham's Lunar Crescent Visibility Criterion

John Knight Fotheringham was born in 1874 in Britain. An expert in historical astronomy, he was also influential in establishing the chronology of the Babylon empire In 1910, Fotheringham published a study on the lunar crescent visibility criterion, incidentally sparking interest on the matter which has been stagnant for at least two centuries. While works of lunar crescent visibility has been published since 1868 by Johann Schmidt, a majority of the studies discuss the report of lunar crescent visibility and lunar crescent visibility criterion of the past, whereas Fotheringham was the first to introduce his own lunar crescent visibility criterion.

Fotheringham incorporated altitude and azimuth in his lunar crescent visibility criterion (Fotheringham, 1910). His curve can roughly be represented in the expression of,

$$Arcv = -0.1758223322\,\Delta AZ + 0.0225942071\,\Delta AZ^2 + -0.0009955850\,\Delta AZ^3 + 12.0825223783$$

3.1

Source: Researcher Data

His curve is calculated at sunset. Fotheringham does not state where he gathered ideas to construct the lunar crescent visibility curve using altitude and azimuth parameter, although Ilyas claimed that it was inspired by the Battani lunar crescent visibility curve (Ilyas, 1987). Fotheringham formulated his lunar crescent visibility curve from 55 positive data and 21 negative data of lunar crescent observation. His data is compiled from the collection of Mommsen and Julius Schmidt lunar crescent visibility data (Mommsen, 1883; Schmidt, 1868). He added that his lunar crescent visibility curve is applicable at any given location, with slight adjustments according to atmospheric extinction. The Fotheringham table of lunar crescent visibility curve is portrayed in Table 3.1 below.

Table 3.2. Fotheringham Table of Lunar Crescent Visibility Curve

Difference in Azimuth (˚)	Arc of Vision
0	12
5	11.9
10	11.4
15	11.0
20	10.0
23	7.7

Source : Fotheringham (1910)

The Fotheringham lunar crescent visibility criterion suggests that the lunar crescent will be visible at a lower altitude if separated by a considerable number of azimuths. Deducing from his visibility curve expression, at 38 degrees of azimuth, lunar crescent is visible at 0 degree of altitude. In real observation, it is not feasible for lunar crescent to be visible at 0 degree of altitude due to the effect of concentrated air mass and high level atmospheric extinction (Schaefer, 1986).

Maunder criticized the Fatoohi visibility curve, stating that his design is primarily based on positive lunar crescent visibility records and ignored a majority of negative sightings. Maunder adds that if a lunar crescent is reported visible at a given parameter at a certain location, it does not guarantee that the same lunar crescent parameter would bevisible at other times and other locations. This is because cloud and atmospheric conditions could hamper the visibility of a lunar crescent. Fotheringham's lack of attention to negative lunar crescent observation has caused its visibility curve to be located at a higher visibility threshold, consequently causing the inability to accurately predict several negative lunar crescent visibility reports (Fatoohi tand Stephenson, F. Richard; Dargazelli, 1999).

Fotheringham data compilation was carried out based on two sources, Mommsen, and Schmidt, both located in Athens. This made his visibility only viable for Athens, and susceptible to error at other latitudes. His data of altitude and azimuth were calculated without consideration of parallax and refraction, making his data subject to error up to 1 degree in real observation. Fatoohi, when examining Fotheringham's lunar crescent visibility criterion, discovered that Maunder's claim on the flaws of Fotheringham's lunar crescent visibility criterion was true. Fotheringham's high visibility curve ignored most of the negative lunar crescent sightings. In addition, Fotheringham's claim that his criterion is adaptable at any given latitude is erroneous as it was discovered that his criterion has been highly erroneous in predicting lunar crescent visibility at other latitudes (Fatoohi & Stephenson, F. Richard; Dargazelli, 1999).

DOI:https//doi.org/10.22452/mjs.vol44no1.12

Despite its shortcomings, the Fatoohi lunar crescent visibility criterion has sparked positive competition among astronomers in designing lunar crescent visibility criterion. His altitude and azimuth parameter led to the design of Maunder lunar crescent visibility criterion, and inspired research on other topocentric parameter of lunar crescent visibility such as elongation and width, His framework of altitude-azimuth parameter is still being used today by Muslim countries (such as the regional body MABIMS) to determine their new Hijri month (Fatoohi, 1998).

Maunder's Lunar Crescent Visibility Criterion

Edward Walter Maunder was a British resident, born in 1851. He was an influential astronomer in solar observation, famously associated with the term *Maunder Minimum* to describe the period of prolonged solar minimum from 1645 to 1715 (Eddy, 1976). In 1911, Maunder published a lunar crescent visibility criterion in his article "On the Smallest Visible Phase of the Moon". His article is a form of refinement of Fotheringham's works, which he heavily criticised as being too pessimistic in predicting lunar crescent visibility.

Similarly with Fotheringham, Maunder incorporates altitude and azimuth for his lunar crescent visibility criterion. His lunar crescent visibility criterion is demonstrated with a table and can be expressed in the form of a formula, as follows:

$$ArcV = 11 - 0.05\,\Delta AZ - 0.01\,\Delta AZ^2$$

3.1

Source : Researcher Data

The Maunder criterion is calculated at sunset, and the Maunder table of lunar crescent visibility curve is demonstrated in Table 3.2 below.

Table 3.3. The Maunder Table of lunar crescent visibility criterion

Different in Azimuth (°)	Arc of Vision (°)
0	11.0
5	10.5
10	9.5
15	8.0
20	6.0

Source : Maunder (1911)

Maunder applied the same altitude-azimuth framework as Fotheringham for his lunar crescent visibility criterion, the only difference being that the Maunder visibility curve is lower than the Fotheringham visibility curve. This is due to the consideration of negative observation in Maunder visibility curve (Krauss, 2012). The Maunder lunar crescent visibility criterion uses the same data as Fotheringham with an additional 11 data from various latitudes, amassing 87 data from lunar crescent visibility records, with 66 positive observations and 21 negative observations. In terms of location, the Maunder lunar crescent visibility data is clustered around Athens. Maunder insisted that his visibility curve is more reliable since it has considered both negative and positive observations. As the Maunder lunar crescent visibility curve uses the same framework with Fotheringham, his criterion would suggest that a lunar crescent is visible at 0 degree arc of vision and 30 degrees of difference in azimuth, which would be impossible due to atmospheric extinction and air mass. Alternatively, the Maunder visibility curve suggests that the lunar crescent is visible at 11.0 degrees arc of vision, whilst its azimuthal difference is 0 degree. This is not necessarily the case, as Fatoohi recorded a visible lunar crescent at as low as 6.2 degrees and 0.5 azimuthal difference (Fatoohi et al., 1998).

Maunder's correction of the Fotheringham visibility curve, although commendable, is still under the framework of altitude-azimuth visibility curve. Despite Maunder's attempt to accurately draw the line between positive and negative lunar crescent, his altitude-azimuth criterion has produced the sames issue as that which arose for the Fotheringham criterion. A high visibility curve would favour prediction for negative data but it would be unable to predict positive data, while attempts to lower the visibility curve will favour prediction for positive data but reduce the successful rate of negative data prediction. It is also noted that the framework of altitude-azimuth lunar crescent visibility criterion is dependent on latitude.

Fatoohi, in his assessment of Maunder's visibility curve accuracy, found that it has a contradiction rate of 17.8 percent in predicting positive observation, while 15.5 percent of the negative lunar crescent observation data was found to fall above Maunder's visibility curve. Thus, Maunder visibility curve can be said to be far superior to other visibility curves that adopt the altitude-azimuth framework, such as Neugebauer and Schoch. However, due to its adoption of the altitude-azimuth framework itself, its capability to predict lunar crescent visibility is not satisfactory (Neugebauer, 1929). Maunder's lunar crescent visibility criterion, despite its flaws, demonstrated how lunar crescent visibility is designed. Maunder showed that rather than favouring positive observation in the construction of lunar crescent visibility criterion, the consideration of negative observation greatly increases the accuracy of any criterion.

Danjon's Lunar Crescent Visibility Criterion

André-Louis Danjon was born in Caen, France in 1890. He was a notable French astronomer, famously credited for introducing a qualitative scale for measuring appearance and luminosity of the lunar crescent, known as the Danjon scale. In 1936, through a collection of 75 measurements and estimation of lunar crescent lengths, Danjon published a work entitled "Le Croissant Lunaire", or "The Lunar Crescent". In his work, Danjon explained the relationship between angle of separation from sun and moon or elongation against the length of a lunar crescent. Danjon stated that the length of lunar crescent increases from 0 degree to 180 degrees in proportion with elongation from 7 degrees to 180 degrees. He deduced that the lunar crescent is invisible for elongations below 7 degrees due to being shadowed of the lunar mountain. Danjon's compilation of 75 measurements of lunar he crescent length showed elongation ranging from 8 degrees to 90 degrees. This indicates that the value of 7 degrees is not the result of direct measurement of lunar crescent, instead it is a product of extrapolation from his graph. In fact, the lowest crescent length was recorded at 6.2 degrees with 8 degrees of elongation. This means that the result obtained of 7 degrees is an interpretation by Danjon and subject to other interpretations by other researchers (Danjon, 1936).

The limit of 7-degree elongation for crescent length, or currently known as the Danjon limit, has been highly contested by researchers. McNally argued that the average lunar radius has a variation of 0.6%, which is too small to cast a shadow that overcast lunar crescent length (McNally, 1983). He then explained that the deficiency of lunar crescent length is due to atmospheric seeing on the cusp of the lunar crescent. Atmospheric seeing cause the cusp brightness of the lunar crescent to be reduced, hence impact the shortening of the visible cusps of lunar crescent. Schaefer (1991) on the other hands, provided a different explanation of the shortening of the lunar crescentClick or tap here to enter text.. He agreed with McNally that it is not plausible to attribute lunar crescent shortening to the shadow of lunar mountain, as it requires a height of 12 km of lunar mountain shadow to overcast the lunar crescent. The highest mountain on the moon is Mons Huygens that has 5.5 km in elevation (Spain, 2009). However, Schaefer disagrees with McNally on the causative effect of atmospheric seeing on the length of lunar crescent. Schaefer supplemented his disagreement by stating that his Moonwatch project indicates that both telescopic and visual observations report the same length of lunar crescent. By McNally modelling, telescopic and visual observers should have different impact of atmospheric seeing, thus contributing to different lengths of lunar crescents (L. Doggett et al., 1994; Ilyas, 1983b). Schaefer conceded that McNally modelling is not applicable in explaining the length of lunar crescent. Schaefer suggested that the reason of the shortening length of lunar crescent at lower elongation is due to the sharp reduction of integrated brightness towards the cusps. The reduction in brightness decreases the detectable contrast, thus contributing to the shortening of lunar crescent. Schaefer then cemented his theory that a 7.5 degree of elongation would be a plausible elongation limit for detectable lunar crescent, or Danjon limit.

Agreeing with Schaefer, Ilyas (1983b) conceded that the shortening is due to the brightness deficiency at the cusps, making it undetectable in human eye Click or tap here to enter text.·. However, Ilyas provided a different model to explain his theory, where he eliminated the 8-degree elongation data in Danjon measurement, and then provide a new extrapolation curve that has the lowest limit of 10.5 degree in elongation. He also supported his argument for 10.5 degrees in elongation by deriving the value of elongation from the lowest limit of width. He argued that the lowest limit of detectable width w= 0.25’ would attribute to elongation of 10.5 degrees using the formulation $W = dsin^2(\frac{L}{2})$.

Sultan (2007) attempted to provide a different explanation for the shortening of lunar crescent lengthClick or tap here to enter text.. He argued that lunar crescent visibility is dependent on the surface brightness per area of the lunar crescent, instead of total integrated brightness. This means that the absence of lunar crescent data for elongation below 7.5 degrees is due to the surface brightness of the lunar crescent at the cusps having a low contrast to be visible with the naked eye. However, optical aided observation is a different case. Sultan argued that optical aided observation would be able to break the 7-degree Danjon limit as optical aided observation can increase the size of the lunar disk while maintaining its surface brightness. Sultan then proved theoretically that optical aided observation at 200 magnifications is able to observe a lunar crescent at 5-degree elongation.

Hasanzadeh (2012) conducted a multi-test to examine the Danjon limit of lunar crescent visibility. Amir conducted experiments which involved extrapolation of elongation against the length of lunar crescent, with additional parameter of atmospheric seeing, lunar mountain shadowing and libration. Amir also experimented with Sultan’s method of determining Danjon limit, by observing the lunar crescent at 120 magnifications. Interestingly, all the experiments conducted by Hasanzadeh resulted in the limiteds elongation of 5 degrees for lunar crescent visibility.

Despite the differences in explaining the reasons for lunar crescent length shortening, Danjon, McNally, Schaefer, Ilyas, Sultan and Hasanzadeh have all contributed to understanding the limits of lunar crescent visibility. Danjon and Schaefer were correct to predict the visibility limit at 7 – 7.5 degree elongation, as it is proven that the naked eye is capable of detecting lunar crescent at 7.7 degree of elongation. The argument contended by McNally, Sultan, and Amir, that lunar crescent is possible to be sighted at elongation below 7 degrees, is warranted, with the 6.0 degree and 3.5 degree of elongation being applicable for optical aided and CCD observation. The claim for 10.5 degree elongation for naked eye limit by Ilyas is somewhat justified with a majority of the lunar crescent visibility falling under the range of 9.0 to 10.5 degree of elongation.

Bruin’s Lunar Crescent Visibility Criterion

Frans Bruin was born in 1922, in Hague, Netherlands. He was a professor of physics in American University of Beirut and director of Universität Bern Astronomical Institute observatory. He was one of the famous historians of astronomy and had the opportunity of working together with the likes of Otto Neugebauer and Edward Kennedy (King, 2002). In 1977, Bruin constructed a lunar crescent visibility criterion that pioneers in inclusion of astrophysical aspects of lunar crescent visibility. Bruin incorporates the parameter of lunar width, altitude, and azimuth in his criterion, which was expressed in the various values of width ranging from 0.5’,0.7’,1’, 2’ and 3’, with attribution of solar depression and arc of vision on its axis. The Bruin visibility curve s as portrayed in Table 3.3 below (Bruin, 1977).

Table 3.4. Bruin Tables of Value for Lunar Crescent Limiting Visibility

Lunar Width (‘)	Arc of Vision (°)
0.5	8.45
0.7	7.23
1	6.55
2	5.05
3	4.77

Source : Bruin (1977)

The application of Bruin lunar crescent visibility criterion is complicated. First, the width of the observed lunar crescent needs to be calculated. Taking for example a lunar crescent width of 2’, during lunar crescent observation, at 5.5 degrees of lunar altitude, the lunar crescent is visible at solar depression of 4.0 degrees until 0.8 degrees, meaning that it has a 12.8-minute window of opportunity. Bruin’s lunar crescent visibility criterion is not only able to predict the visibility of the lunar crescent, at the same time it is also able to estimate the time windows for successful observation. The Bruin lunar crescent visibility criterion is expressed in Equation 3.3.

$$ArcV = -0.1324039674w + 0.0009057913w^2 + -0.0000021108\,w^3 + 11.5621745317 \quad 3.2$$

Source : Researcher’s Data

In designing his criterion, Bruin has made the following assumption. First, he assumed that the sky brightness is uniform regardless of altitude and azimuth, with only solar depression as a single brightness variable (Koomen et al., 1952). Second, Bruin assumed that the brightness of the lunar crescent is uniform across its surface, with only the lunar crescent altitude acting as a presenter for atmospheric extinction. Third, Bruin assumed that the minimum contrast required for lunar crescent visibility is associated with lunar surface area. For this assumption, Bruin adopted the works of Siedentopf circular disk visibility threshold and converted it into lunar width (Bemporad, 1904). Bruin used the assumption in his design for lunar crescent visibility criterion, instead of using actual observation of lunar crescent. Bruin stated that his criterion has been experimented on for 10 years, and his assumptions are correct without requiring further refinement.

All three of Bruin’s assumptions were in fact incorrect. First, the assumption that sky brightness is uniform with only solar depression acting as a single brightness variable is entirely wrong. The model developed by Kastner demonstrated that the brightness of sky during twilight is dependent on solar depression, altitude, and azimuth of the observed object (Kastner, 1976). The Kastner modelling warrants a high accuracy and is still relevant for current application (Faid et al., 2016, 2018).

Second, the assumption that the brightness of the lunar crescent is singularly dependent on lunar crescent altitude is not entirely correct. Although lunar crescent altitude can represent atmospheric extinction in the simplest form, the impact of atmospheric extinction to lunar brightness is more complex and require complex variables. Schaefer has laid out the computations required to measure the impact of atmospheric extinction on lunar brightness, encompassing air mass, temperature, season, atmospheric layer, humidity, altitude, latitude, and wavelength. Thus, to simply express the impact of atmospheric extinction on lunar brightness in the form of lunar crescent altitude is an oversimplification. Third, Bruin adopted Siedentopf circular disk visibility threshold in his criterion by assuming its applicability for lunar crescent visibility threshold. Circular disk visibility and lunar crescent visibility threshold are heterogenous. This is because the surface area and the shape of lunar crescent are entirely different from that of circular disk. Blackwell's model of crescent visibility threshold in 1946 is more suitable for Bruin lunar crescent visibility criterion instead of Siedentopf’s works (Blackwell, 1946).

Fatoohi, in assessing the reliability of Bruin’s lunar crescent visibility criterion, discovered that Bruin has underestimated the capability of the human eye to detect the limiting width of lunar crescent (Fatoohi, 1998). There are 77 reports of lunar sightings with lunar crescent width of less than 0.5’ observed by the naked eye, with the thinnest width to be at 0.17’. This is way below the visibility limit of the Bruin lunar crescent width of 0.5’. Fatoohi further added that Bruin that miss predict 27.7% of the positive observation, 9.6% of negative observation. The Bruin lunar crescent visibility criterion, despite the incorrect assumption of lunar crescent visibility and its underestimation of the human eye detection capability, was a pioneer in designing an astrophysical lunar crescent visibility criterion. It created a pathway for Schaefer, Sultan, and Faid to create their own astrophysical lunar crescent visibility criterion (Bradley Schaefer, 1996a; Sultan, 2007b; Muhamad Syazwan Faid, Nawawi et al., 2023).

Ilyas’s Lunar Crescent Visibility Criterion

Ilyas was born in Meerut, India, in 1950. He is one of the Muslim pioneers in the research of lunar crescent visibility. Between 1983 and 1994, he published at least 10 articles on lunar crescent visibility and Islamic calendar. Ilyas played a pivotal role in bringing Muslim astronomers into research of lunar crescent visibility and Islamic Calendar, where during his time, a majority of these endeavours were carried out by Islamic Scholars without prior scientific or astronomical knowledge. His work on lunar crescent visibility and Islamic Calendar sparked the interest of other Muslim astronomers to study and examine the reliability of lunar crescent visibility criterion (Mohammad Ilyas, 1986).

Ilyas has produced various lunar crescent visibility criteria, moon age-latitude lunar crescent visibility criterion, lag time-latitude lunar crescent visibility criterion, lunar crescent altitude and sun-moon azimuth, revision of Danjon limit, revision of Bruin lunar width, arc of light and arc of vision. His primary lunar crescent visibility criterion is altitude-azimuth criterion as portrayed in Table 3.4 and Equation 3.5.

Table 3.5. Ilyas' Lunar Crescent Table Data

Azimuth Difference (˚)	Arc of Vision (˚)
0	10.3
5	9.9
10	9.15
15	7.9
20	6.4
23	5.6

Source: Ilyas (1994)

$$Arcv = -0.0027356815\,DAZ + -0.0136648716\,DAZ^2 + 0.0002119205\,DAZ^3 + 10.2832719598 \quad 3.3$$

Source: Researcher's Data

Ilyas suggested that the limiting elongation for lunar crescent visibility is 10.5 degrees, which is 3.5 degrees more than Danjon limit, while the limiting altitude for lunar crescent visibility is 10 degrees. Ilyas derived the lunar crescent altitude values from Maunder lunar crescent visibility criterion where Maunder's limiting threshold of lunar crescent altitude at 0 degree of azimuth is 11.0 degrees (Maunder, 1911). The 10.5 elongation limit is derived from reextrapolation of Danjon graph and a reinterpretation of Bruin limit of lunar width. Ilyas discovered that if the Bruin lunar width limit is lowered to 0.25, it would correspond to the geocentric elongation of 10.5 degrees. Ilyas claimed that the drawing of his lunar crescent visibility graph is a combination of Bruin's and Maunder's lunar crescent visibility criterion. Ilyas further cemented that his criterion is agreeable at any given latitude and contains a small value of uncertainty.

Fatoohi (1998) argued that Ilyas has suffered a fundamental flaw in his design of lunar crescent visibility criterion. Maunder's and Bruin's lunar crescent visibility criterion are not related to one another. Maunder derived his lunar crescent visibility criterion from 91 data of lunar crescent visibility, while Bruin designed his lunar crescent visibility criterion using theoretical value of sky brightness, lunar crescent illumination, and contrast threshold. Ilyas also did not make any attempt to demonstrate how the combination of two independent lunar crescent visibility criteria works. In addition, it is stipulated by Fatoohi that the Maunder lunar crescent visibility criterion does not work at all latitudes, whilst the Bruin lunar crescent visibility criterion has an extensive range of uncertainty. In addition, McPartlan commented that the Ilyas elongation limit to underestimation of human eye should be lowered by 0.5 degrees to account for the number of positive lunar crescent observations that fall under Ilyas' invisibility line (McPartlan, 1996).

Fatoohi discovered that the Ilyas altitude-elongation criterion has 29.8 percent contradiction rate in predicting invisibility, and 7.8 percent contradiction rate in predicting visibility. The Ilyas altitude-azimuth lunar crescent visibility criterion was unable to predict 28.6 percent of negative sighting, and 11.3 percent of positive sighting. His limiting value of elongation is also not dependable as the lunar crescent was found to be detectable at 7.7 degrees in elongation. Despite the flaws in his design of lunar crescent visibility criterion, Ilyas has shown there to be various possible presentations in designing lunar crescent visibility criterion. Ilyas was also perhaps the most influential astronomer in lunar crescent visibility research among Muslims.

Schaefer's Lunar Crescent Visibility Criterion

Bradley Schaefer was a Professor Emeritus in Louisiana State University. He was awarded with Nobel prize and Gruber prize for his team's discovery of Dark Energy. In 1983, Schaefer embarked on a journey of lunar crescent visibility research. In his publication on lunar crescent visibility entitled, "Algorithm of Lunar Crescent Visibility", he criticized the current lunar crescent visibility criteria to be limited to geometrical measurement, whereas visibility is a problem that involves atmospheric and human eye sensitivity (Schaefer, 1987). He proposed that the lunar crescent visibility criteria should be designed with consideration of physical, meteorological and physiological equation, the same framework that Bruin has designed for his lunar crescent visibility criterion. To achieve his goal of comprehensive lunar crescent visibility criterion, Bradley Schaefer launched the Moonwatch project, an open project for lunar crescent observation. Through the project, Schaefer was able to gather data of lunar crescent visibility through instrumentation, physiological and atmospheric perspective (L. Doggett et al., 1994; L. E. Doggett et al., 1988; L. Doggett & Schaefer, 1989). He also conducted a study on lunar brightness (Krisciunas & Schaefer, 1991; Schaefer, 1990), twilight sky brightness (Schaefer, 1987), atmospheric extinction (Schaefer, 1986), telescopic limiting magnitude (Schaefer, 1990), lunar physical observation (Schaefer, 1991), and visibility threshold (Schaefer, 1998) to further refine his algorithm. Schaefer tested his algorithm on sunspot visibility (Schaefer, 1991) and validated the date of Jesus crucifixion (Schaefer, 1990). In 1993, Schaefer produced his computation formula for his algorithm (Schaefer, 1993), and in 2000, 12 years later, he updated the final version of his algorithm computation formula (Schaefer, 2000).

Numerous researchers have tried to emulate Schaefer's lunar crescent visibility algorithm. Fatoohi (1998) commented that Schaefer never actually published the full version of his formulation, despite comparatively assessing his algorithm against other lunar crescent visibility criteria. Ilyas (1994, 8) highlighted that Schaefer's lunar crescent visibility algorithm was not published to the public during his time. He also added that Schaefer algorithm are too complicated and not practical for long-term prediction, particularly in application of Islamic calendar determination. Yallop (1998), in agreement with Fatoohi and Ilyas, also noted that it was difficult to replicate Schaefer's calculated algorithm as the information in regards to the algorithm is conflicting between his papers. It is not until recently that a Muslim astronomer, Faid, Nawawi et al. (2023) was able to emulate Schaefer's lunar crescent visibility algorithm. Faid et al., commented that most of the Schaefer visibility algorithms are available in his publication "New Method for Archeoastronomy", while pieces of formula are scattered in other Schaefer publications. Twilight sky brightness formulation can be collected from the "Heliacal Sky Rise" paper (Schaefer, 1987,11), the model of lunar crescent brightness can be collected from "Model of Moon Brightness" paper, telescopic visibility threshold can be collected from "Telescopic Limiting Magnitude" paper (Schaefer, 1990) and atmospheric extinction can be collected from "Atmospheric Extinction effect on stellar Alignment" paper. Faid also highlighted that Schaefer did not provide unit for most of his contrast formulations, making it even more difficult and requiring trial and error (Faid et al., 2024).

Due to its difficulty and unavailability, the reliability results of Schaefer's lunar crescent visibility algorithm is not yet evaluated by any researcher. In partial assessment of Schaefer's algorithm, Loewinger discovered that Schaefer miscalculated the lag time value of his data (Loewinger, 1995). Sultan also found that Schaefer has a confusing definition of lunar brightness, frequently interchanging the definition of integrated brightness and surface brightness (Sultan, 2004,11). Schaefer's algorithm, despite its flaws, indicate that the approach to convert the computation of lunar crescent visibility into a full theoretical formulation is entirely plausible.

Yallop's Lunar Crescent Visibility Criterion

In 1988, Bernard Yallop published a lunar crescent visibility criterion that can be categorized into ranges of visibility. Yallop formulated his lunar crescent visibility criterion by adapting arc of vision and sun-moon azimuth into q-value expression, where w is the topocentric lunar width. His arc of vision and sun-moon azimuth was not calculated at sunset, as done by his predecessor. According to him, this is because lunar crescent is not sighted during sunset, instead it is visible after the contrast between sky and lunar brightness can adequately be seen by observer, which would be during the minutes after sunset. The value is calculated during this best time, which Yallop expressed in Equation (3);

$$Best\ Time = Time\ of\ sunset + \frac{4}{9}lagtime \qquad (4)$$

Yallop categorized the values of q into ranges of lunar crescent visibility, which are: Easily Visible, Visible Under Perfect Condition, May Need Optical Aid to Find Crescent, Will Need Optical Aid to Find Crescent, Not Visible with a Telescope, and Not Visible. The categorization is as set out in Table 6.

Table 6. Yallop Lunar Crescent Visibility Criterion

Criterion	Range	Remarks
(A)	$q > 0.216$	Easily visible, ARCL> 12 °
(B)	$0.216 \geq q > -0.014$	Visible under perfect conditions
(C)	$-0.014 \geq q > -0.160$	May need optical aid to find crescent
(D)	$-0.160 \geq q > -0.232$	Will need optical aid to find crescent
(E)	$-0.232 \geq q > -0.293$	Not visible with a telescope, ARCL < 8.5°
(F)	$-0.293 \geq q$	Not visible, below Danjon limit, ARCL<. 8°

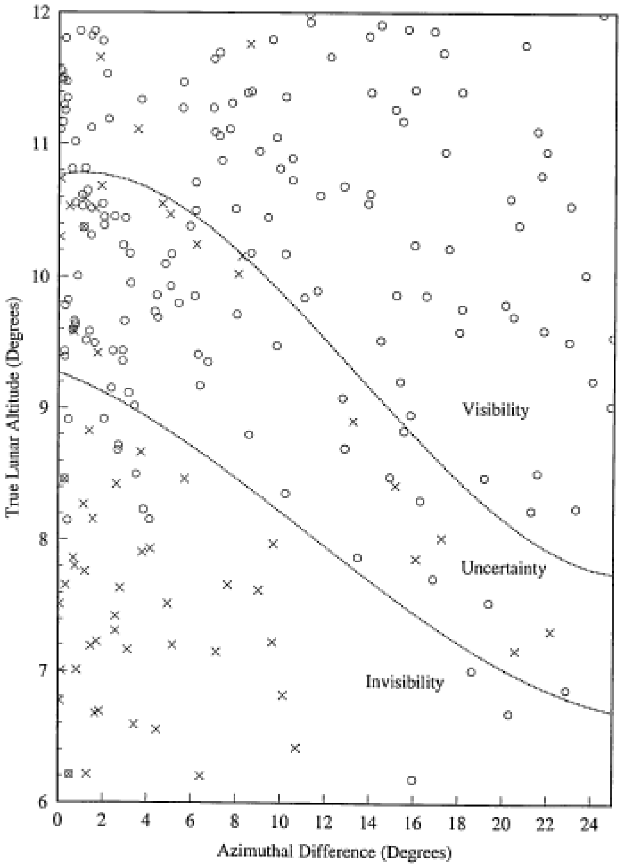
Yallop based his *q*-formulation from Indian lunar crescent visibility criterion, and Neugebauer lunar crescent visibility criterion (Fatoohi, 1998, 124; Neugebauer, 1929, 111). Yallop's categorization of visibility ranges was based on 295 records of lunar crescent observation compiled by Schaefer and Doggett. Fatoohi commented that Yallop's categorization, particularly the E & F categories, were not needed as both can be counted as invisible sighting. Fatoohi added that the Neugebauer and Bruin lunar crescent visibility criterion was not dependable enough to function as a basis for Yallop lunar crescent visibility criterion. Fatoohi also argued that Yallop's categorization was not balanced as he found that from 295 data of lunar crescent sighting, 166 fall under category A, while only 68,26,14,4 and 17 fall under categories B, C, D, E and F, respectively. These disparities indicate that more data is needed for each category to validate the ranges of visibility (Fatoohi, 1998). In examining Yallop's criterion, Fatoohi found that there was a high number of errors, except for category A. Fatoohi opined that Yallop's criterion was highly unreliable as it was unable to accurately predict the lunar crescent visibility.

Fatoohi also criticised Yallop's lunar crescent visibility criterion for its adoption of inconsistent visibility ranges, and weak mathematical foundation. Despite its weakness, Yallop is known for popularizing the concepts of visibility ranges and best times, which then influences researchers like Qureshi and Sultan to produce their definition of visibility ranges and best times, respectively (Qureshi, 2010; Sultan, 2006).

Fatoohi's Lunar Crescent Visibility Criterion

Fatoohi's PhD thesis was entitled "First visibility of the lunar crescent and other problems in historical astronomy". In his thesis, Fatoohi (1998) suggested an altitude-azimuth lunar crescent visibility criterion as in Equation (9). The lunar crescent altitude located in between $ArcV_{Upper\ Limit}$ and $ArcV_{Lower\ Limit}$ is placed at a zone of uncertainty. Fatoohi's lunar crescent visibility criterion graphs can be portrayed as in Figure 2 below.

Figure 1. Fatoohi's Lunar Crescent Visibility Criterion



Source : Fatoohi, L, First visibility of the lunar crescent and other problems in historical astronomy, 1988, Durham University, 141

Fatoohi incorporated Ilyas' idea of zone of uncertainty into his design of lunar crescent visibility criterion. While Ilyas' idea of zone of uncertainty refers to the geographical latitude-longitude where the lunar crescent cannot be accurately predicted using his criterion in the International Lunar Date Line (Ilyas, 1997), Fatoohi incorporated the zone of uncertainty directly in his formulation of lunar crescent visibility criterion, where the visibility of the lunar crescent cannot be predicted accurately. Fatoohi noted that both the upper limit and lower limit of his criterion only has error of 5.9 percent in predicting negative lunar crescent sighting, and 3.6 percent in predicting positive lunar crescent sighting. However, in between both limits, the accuracy of lunar crescent prediction falls rapidly. In his test, Fatoohi found that the zone of uncertainty in his criterion can accommodate 27.4 percent of negative sighting error, and 16.4 percent of positive sighting error.

Fatoohi also noted that the implementation of his lunar crescent visibility criterion is more applicable for the determination of new Hijri month, where both the practice of astronomical calculation and lunar crescent sighting can be adopted. If the lunar crescent is located above the upper limit of the Fatoohi lunar crescent visibility criterion, then it can be confidently expected that lunar crescent will be sighted, and new Hijri month is commenced. If the lunar crescent is located below the lower limit of the Fatoohi lunar crescent visibility criterion, then it can be confidently expected that the lunar crescent will not be sighted, and the current Islamic month will continue until the 30th day. However, if the lunar crescent is located in between the upper limit and the lower limit, the lunar crescent needs to be sighted to confirm its visibility. Fatoohi noted that in his test on 300 months of Islamic Calendar in Mecca, Baghdad and Casablanca, there were around 7 percent of cases where the lunar crescent was located in the zone of uncertainty.

The idea of using lunar crescent sighting for zone of uncertaintyr looks practical on paper and relatable to the practice of the Prophet. However, in real cases, as discussed in the previous chapter, some countries have limitations in exercising lunar crescent sighting to determine the new Hijri month, making the idea of incorporating the zone of uncertainty on lunar crescent visibility criterion not feasible for real time application. The incorporation of zone of uncertainty also limits the usage of the lunar crescent visibility criterion for historic calendrical dating purposes, as lunar crescents located in the zone of uncertainty cannot be accurately calculated.

Odeh's Lunar Crescent Visibility Criterion

Mohamad Shaukat Odeh was born in Kuwait in 1979. He is a member of the Arab Union for Astronomy and Space Science. He is renowned for founding the global lunar crescent observation project, called the Islamic Crescent Observation Project, since 1998. The project has since then collected more than 2000 data of lunar crescent sighting worldwide. Odeh is also involved in the development of Accurate Time, an Islamic astronomy software that can function as a lunar crescent visibility calculator, Islamic calendar calculator, and prayer time calculator. He is one of the most influential lunar crescent sighting astronomers in our time.

Odeh published a lunar crescent visibility criterion in 2005. His lunar crescent visibility criterion is categorized into ranges of visibility, similar with Yallop's lunar crescent visibility criterion. Odeh's lunar crescent visibility criterion has 4 categories of visibility, with values to determine the ranges of visibility as set out in Table 7.

Table 7. Odeh's Lunar Crescent Visibility Criterion

Category	Remarks	Criteria
Zone A	Crescent visible by naked eye	$V \geq 5.65$
Zone B	Crescent visible by optical aid, and might be visible by naked eye	$2 \leq V < 5.65$
Zone C	Crescent visible by optical aid	$-0.96 \leq V < 2$
Zone D	Crescent is not visible by optical aid	$V < -0.96$

Odeh determine his V value using the same formula as Yallop used in determining his Q value. Odeh lowered his arc of vision threshold from 11.8371 degrees to 7.1651 degrees. Odeh lunar crescent visibility criterion is formulated through a compilation of

737 records of lunar crescent sighting. These include 294 records of lunar crescent sighting from Schaefer's list, 6 records from Jim Stamm, 42 records from South Africa Astronomical Observatory, 15 records from Mohsen Mirsaeed, 57 records from Alireza Mehrani and 323 records from ICOP. As Odeh follows Yallop's lunar crescent visibility criterion, it has the same weakness as the Yallop criterion. Although the Odeh visibility classification is more practical than Yallop's visibility classification, the number of lunar crescent sightings on each visibility group is not balanced with 46 lunar crescent sightings on Group D, 117 lunar crescent sightings on Group C, 255 lunar crescent sroup B, and 160 lunar crescent sightings on Group A. Furthermore, the ratio of positive to negative sightings on each group was found to be lopsided, with Group A having a majority of negative lunar crescent sighting, and group D having a majority of positive sighting. The contradiction rate analysis of Odeh's lunar crescent visibility criterion has not been examined by any scholar so far.

Qureshi's Lunar Crescent Visibility Criterion

Mohamad Shahid Qureshi is a Pakistan-born astronomer, mathematician, and astrophysicist. He is a former director and Professor at the Institute of Space and Planetary Astrophysics, Karachi Universiti, Pakistan. Shahid Qureshi's Doctoral thesis is entitled "Earliest Visibility of Lunar Crescent" making him an expert in designing lunar crescent visibility. Shahid Qureshi has published at least five papers concerning the visibility of lunar crescent in Pakistan.

Qureshi produced his own lunar crescent visibility criterion in 2010. Shahid Qureshi's criterion is in a similar framework with Yallop's and Odeh's criterion, which adapt ranges of lunar crescent visibility. Qureshi's ranges of visibility are categorized into Easily Visible, Visible Under Perfect Condition, May Require Optical Aid to Find Crescent, Require Optical Aid, and Not Visible with Optical Aid. His table for visibilities ranges is expressed ina Table 8.

Table 8. Qureshi Lunar Crescent Visibility Criterion

Visibility	Criteria
Easily Visible (EV)	s-value > 0.15
Visible under perfect conditions (VUPC)	0.05 < s-value < 0.15
May require optical aid to find crescent (MROA)	-0.06 < s-value < 0.05
Require optical aid (ROA)	-0.16 < s-value < -0.06
Not visible with optical aid (I)	s-value <-0.16

Qureshi's visibility ranges are based on s value. The s value takes the same arc of vision and width parameter used by Yallop and Odeh, however Qureshi changed the coefficient to fit with his data. Qureshi's data were calculated using a website which claimed to have adapted Schaefer's visibility logarithm. The website is, however, inaccessible to verify the computation. Qureshi highlighted that his s value is more accurate, as it considers the brightness of the sky, lunar crescent illumination and detectable contrast threshold. However he does not demonstrate howthe formulation of the S value is conducted. As Qureshi shares the same lunar crescent visibility criteria style as Yallop and Odeh, it does face the issue of unbalanced lunar crescent sighting data of each visibility group. The contradiction rate analysis of Qureshi lunar crescent visibility criterion has not been examined by any scholar so far.

Caldwell's Lunar Crescent Visibility Criterion

John Caldwell is an astronomer who graduated from California Institute of Technology in 1974. In 1979, he obtained his PhD from Princeton University. Previously he was a research fellow on the South African Astronomical Observatory, Cape Town, South Africa. In 2012, John Caldwell published a study on lunar crescent visibility criterion, on the Monthly Notes of the Astronomical Society of South Africa Journal. To date, John Caldwell's lunar crescent visibility criterion is the only published works from African countries on lunar crescent visibility criterion.

Caldwell incorporated moonset-sunset lag time and arc of light in his lunar crescent visibility criterion. Caldwell argued that lag time is a better parameter to determine lunar crescent visibility, as it is applicable at various degrees of latitude. This is because lag time is correlated to the separation angle between the sun and the moon, in contrast to altitude, where it is highly correlated to the local horizon. Lag time is also best paired with elongation as both do not have linear relationship to each other. Caldwell's lunar crescent visibility criterion is expressed in Table 9 below (Caldwell, 2011).

Table 9. Caldwell's Lunar Crescent Visibility Criterion

Criteria	Expression
A . Possible for Naked Eye Sighting	$lag\ (') > -0.9709\ arcl + 44.65$
B. Possible for Optical Aided Sighting	$-0.9709\ arcl + 44.65 > lag\ (') > -1.9230\ arcl + 43.13$
C. Not possible for sighting	$lag\ (') < -1.4150\ arcl + 36.76$

Caldwell's lunar crescent visibility criterion is based on 36 data of positive naked eye and 58 data of positive optical aided lunar crescent sightings from various latitude and longitude. As Caldwell's lunar crescent visibility criterion is based on a small dataset, this makes his criterion susceptible to error. In his graph, Caldwell included negative lunar crescent sighting. The details of negative lunar crescent sighting data are not included in this paper, making reassessment of his criterion to be limited. Lag time parameter was also heavily criticised by Ilyas (1994), and Fatoohi (1998) to be highly unreliable and dependent on latitude. The Caldwell criterion design also suffered from Yallop lunar crescent visibility design, due to unbalanced number of lunar crescent sighting reports on each visibility categorization. The Caldwell lunar crescent visibility criterion is, however, not yet assessed by any modern scholar of lunar crescent visibility criterion.

Lunar Crescent Visibility Criterion

In 2012, Rolf Krauss published a study on lunar crescent visibility using Babylon data of lunar crescent sighting. A 95-page article written by Krauss contains arguments about data validity, interpretation of modern lunar crescent visibility criterion based on Babylon data, effect of weather on lunar crescent visibility, and an azimuth-altitude lunar crescent visibility criterion.

Krauss included the seasonal factor into his criterion, citing the Schaefer visibility logarithm to support his inclusion. However, the inclusion has caused Krauss's criterion to have large deviation error, up to 1.8 degrees. For calendrical purposes, a large deviation error can lead to unreliable lunar crescent visibility criterion. The contradiction rate analysis of Krauss lunar crescent visibility criterion has not been examined by any scholar to date.

Table 3.10. Krauss' lunar crescent visibility criterion

	September to March Babylonian	March to September Babylonian	Athenian
DAZ (°)	ArcV (°)		
0	10.1 ± 1.5	10.8 ± 1.4	10.6 ± 1.8
5	10.0	10.7	10.5
10	9.4	10.1	9.95
15	8.4	9.2	9.0
20	7.1	7.8	7.6
22	6.4	7.1	7.0

Source: Krauss (2012)

$$Arcv^{Sept-March} = 0.0246304381\,DAZ + -0.0100243996\,DAZ^2 + 0.0000590234\,DAZ^3 + 10.1050739464$$

$$Arcv^{March-Sept} = 0.0115318125\,DAZ + -0.0075992122\,DAZ^2 \pm 0.0000258220\,DAZ^3 + 10.8074774331$$

$$Arcv^{Athenian} = 0.0291254840\,DAZ + -0.0098347831\,DAZ^2 + 0.0000475196\,DAZ^3 + 10.5981838905$$

3.5

Source: Krauss (2012)

3.12 Gautschy's Lunar Crescent Visibility Criterion

Gautschy, in 2014, published his work on lunar crescent visibility criterion entitled "On the Babylonian sighting-criteria for the lunar crescent and its implications for Egyptian lunar data" (Gautschy, 2014). The article endeavours to produce a lunar crescent visibility criterion based on Babylon prediction and lunar crescent visibility records, and to utilise the produced criterion in establishing an absolute Egyptian chronology.

Gautschy used Yallop's lunar crescent visibility criterion to validate the Babylon records of lunar crescent sightings. Records that contradict Yallop lunar crescent visibility criterion were recalculated to ensure its accuracy. Inaccurate, or unclear records of lunar crescent sighting, were rejected. Gaustchy argued that Krauss' judgement to design a lunar crescent visibility criterion based on season was not justified, as it was proven that season does not affect visibility of lunar crescent. Gaustchy used parameter of lag time and difference in azimuth, as lag time is insensitive to difference calculation reference, either topocentric or geocentric. Gaustchy also evaluated her criterion using Odeh lunar crescent visibility criterion and found that the results she obtained followed Odeh's visibility prediction.

Gaustchy was able to provide a fresh outlook on lunar crescent visibility criterion based on Babylon's lunar crescent visibility records. While Kraus adopted modern altitude-azimuth lunar crescent visibility criterion, Gaustchy was adamant that lag time parameter, which is the parameter that has been adopted since the Babylonian age, was just as efficient as other topocentric parameters. While she admitted that lag time is dependent on latitude, it is applicable for her research purpose, which is to produce an absolute Egyptian chronology specific for latitudes in Egypt.

Table 3.11. Gaustchy's lunar crescent visibility criterion

DAZ (°)	0	2.5	5	7.5	10	15	20	25
Arc of Separation (°)	8.6	8.55	8.45	8.2	7.8	6.5	5.0	3.3
Lag time (')	34m	34m	34m	33m	31m	26m	20m	13m

Source: Gautschy (2014)

$$Latime = 0.3342328913\,\Delta AZ + -0.0715608980\,\Delta AZ^2 + 0.0009924422 * \Delta AZ^3 + 33.8890455442$$

3.6

Source: Gautschy (2014)

Alrefay's Lunar Crescent Visibility Criterion

Thamer Alrefay is an Assistant Professor from Space Research Institute, King Abdul Aziz City for Science and Technology, Riyadh, Saudi Arabia. He is a member of the Canadian Association for Physicists, and completed his PhD in University of New Brunswick, Canada in 2014 under the subject of space physics. Alrefay's interests are Space Physics, Fireball Observation, and Earth Bow Shock.

In 2018, Alrefay and his fellow researchers at King Abdul Aziz City for Science and Technology published their research on the earliest visibility of the lunar crescent. The research is conducted based on 545 observations of lunar crescent sighting in Saudi Arabia within a duration of 27 years. Alrefay et al. (2018) developed a lunar crescent visibility criterion using width and arc of vision parameter, in a similar fashion with Yallop, Qureshi and Odeh. The Alrefay lunar crescent visibility criterion is classified into two categories, which are naked eye and optical aided observation, as expressed in Equation 3.12 below.

$$ARCV_{Naked\ Eye} > 9.34 - 4.51w + 3.3W^2 - 1.01W^3$$

$$ARCV_{Optical\ Aided} > 7.83 - 4.35w + 3.22W^2 - 1.02W^3$$

3.7

Source : Alrefay (2018)

Alrefay argued that the Yallop lunar crescent visibility criterion, and the Odeh lunar crescent visibility criterion, are not consistent with other lunar crescent visibility data calculation reference. In addition, Yallop and Odeh both adopt a topocentric width parameter, without any argument as to why topocentric parameters would help in determining the limiting visibility of lunar crescents. Alrefay argued that his lunar crescent visibility criterion was based on geocentric lunar crescent parameters, and his criterion is consistent with other lunar crescent visibility data calculation reference. The Alrefay lunar crescent visibility criterion, however, was based on Saudi Arabia data and limited to 595 lunar crescent sightings, while the Odeh lunar crescent visibility criterion was based on 737 worldwide lunar crescent sightings. This makes Alrefay unreliable in determining the lunar crescent visibility outside Saudi Arabia. The contradiction rate analysis of Ahmad's lunar crescent visibility criterion has not been examined by any scholar so far.

Ahmad's Lunar Crescent Visibility Criterion

Nazhatulshima Ahmad is an astronomer from Malaysia, who is currently a Senior Lecturer in the Department of Physics, Universiti Malaya, Malaysia. She is highly experienced in the formulation of procedures to validate lunar crescent sightings and developing criterion of lunar crescent visibility in Malaysia. She is also experienced in research fields of spectroscopy of emission line stars; binary stars; asteroids; observation and imaging techniques in optical regions; spectroscopy, photometry, and astrometry. She is currently a member of the International Astronomical Union.

Ahmad published a lunar crescent visibility criterion in 2020. Ahmad et al. (2020,3) criterion is in similar framework with Yallop, Odeh, and Qureshi criteria, which by adapting ranges of lunar crescent visibility criterion. Ahmad applied a unique approach on parameter of lunar crescent, by using a circular regression model. Ahmad's lunar crescent ranges of visibility is categorized into three categories, which are visible to the unaided eye, may need optical aid, and not visible (Ahmad et al., 2022).

Table 12. Ahmad Elongation and Altitude Lunar Crescent Visibility Criterion

Category	EA-test value	N	Y	Total %	Interpretation
A	[0.0086, 00)	21	52	73 (29)	Visible to the unaided eye
B	[0.00516, 0.0086)	26	9	35 (14)	May need optical aid
C	(-00, 0.0052)	126	20	146 (57)	Not visible

Table 13. Ahmad Elongation and Arc of Vision Lunar Crescent Visibility Criterion

Category	EV-test value	N	Y	Total %	Interpretation
A	[0.0039, 00)	59	21	80 (31)	Visible to the unaided eye
B	[-0.0022, 0.0039)	24	16	40 (16)	May need optical aid
C	(-00,-0.0022)	90	44	135 (53)	Not visible

Ahmad's lunar crescent visibility criterion was expressed into two parameter pairings, namely elongation with altitude and elongation with arc of vision. Ahmad argued that the development of the criteria uses only linear statistical theory, while in fact most of the variables in crescent moon data are measured in degree/radians. However, Ahmad did not demonstrate his lunar crescent visibility criterion capability over linear lunar crescent visibility. In addition, Ahmad's lunar crescent visibility criterion was expressed in a form of complex mathematical expression, and not easily applied for Hijri calendrical purposes. Ahmad also did not provide any expression that can be used for lunar crescent visibility criterion analysis, as it is foundthat the expression provided in the research paper does not correlate to the results in Table 3.20 and Table 3.21. The contradiction rate analysis of Ahmad's lunar crescent visibility criterion has not been examined by any scholar so far.

Country Based Lunar Crescent Visibility Criterion

A country based lunar crescent visibility criterion is a criterion used for the purpose of a country's Hijri calendrical determination. A country-based lunar crescent visibility criterion is usually designed at a lower line of lunar crescent visibility, as it is to ensure that no lunar crescent is sighted below the criterion. In addition, a country-based lunar crescent visibility criterion is usually designed in a conditional style lunar crescent visibility criterion. This is in contrast to research-based lunar crescent visibility criterion such as Alrefay et al. and Gautschy, where they used expression style lunar crescent visibility criterion. It can be deduced that the conditional style lunar crescent visibility criterion is simpler for Hijri calendrical calculation, while equation style lunar crescent visibility criterion requires more computation power and advanced programming technique to calculate the Hijri calendar, particularly computations that involve long years of Hijri calendar.

Saudi's Lunar Crescent Visibility Criterion

Saudi Arabia houses the Muslim pilgrimage location, which are Mecca and Madinah. Muhamad was also born in Mecca, and Islamic expansion has Mecca and Madinah as its epicentre. This makes Saudi Arabia the most influential country among the Muslim community. Due to this, several countries follow closely Saudi Arabia's date of Hijri month. Dates of religion importance, such as the day of Arafah, and Eid Adha, impact Muslims worldwide as it relates to their religion practices. This makes a number of countries follow Saudi Arabia lunar crescent visibility criterion, such as Afghanistan, Albania, Algeria, Armenia, Austria, Azerbaijan, Bahrain, Belgium, Belgium, Bolivia, Bulgaria, Burkina Faso, Chechnya, Denmark, Finland, Georgia, Hungary, Iceland, Iraq (Sunnis), Italy, Japan, Kazakhstan, Kuwait, Kyrgyzstan, Lebanon, Mauritania, Palestine, Philippines, Qatar, Romania, Russia, Sudan, Sweden, Switzerland, Syria, Taiwan, Tajikistan, Tatarstan, Togo, Turkmenistan, U.A.E, and Uzbekistan.

As a large number of countries follow Saudi Arabia's lunar crescent visibility criterion, their early criterion is based on the Greenwich time zone. Saudi Arabia's old lunar crescent visibility criterion is simply conditioned as: new Hijri month begins when, after a moon conjunction, sunset occurs before moonset. The old criterion did not consider altitude, age and elongation (Mostafa, 2005). The old criterion was contested by Kordi (2003, 2), as it was not based on Saudi Arabian time zone, or any Saudi Arabian location reference point. In 2000, a new lunar crescent visibility criterion for Saudi Arabia is introduced. This is in conjunction with the new Umm al-Qurra calendar. The criterion condition is as follows:

a. The position of lunar crescent and sun is computed using the Holy Kaaba as reference for calculation.
b. If a lunar crescent is set before sunset during conjunction, an observation is conducted a day after.
c. If a lunar crescent is set after sunset, and its sighting is accepted in accordance with Islamic Jurisprudence of Saudi Arabia.

Mostafa (2005) stated that the new criterion is based on the capability of a lunar crescent sighting by an observer, rather than the capability of a lunar crescent sighting based on a criterion parameter. He added that this reduces errors in lunar crescent report from 14 percent for old criterion into 0 percent for the new criterion. However, as the new Saudi Arabia lunar crescent visibility criterion is solely based on moonset after sunset, there is still room for error in lunar crescent reporting. The world record for lag time is 30 minutes for naked eye, and 20 minutes for optical aid observations. Should a lunar crescent be observed below the world record limit, then the lunar crescent is highly contestable and should be rejected. However, based on Saudi Arabia's new lunar crescent visibility criterion, a lunar crescent sighting is accepted regardless of whether its lag time challenges the world record or not.

Turkey's Lunar Crescent Visibility Criterion

Although it does not have the same magnitude of influence as Saudi Arabia does over Muslim communities worldwide, Turkey has influence in the realm of lunar crescent visibility criterion. Turkey was the first to introduce a lunar crescent visibility criterion in 1978, earlier than Ilyas' series of lunar crescent visibility criteria (Mohd Nawawi et al., 2015). Turkey's1978 lunar crescent visibility criterion is the criterion that is believed to have inspired Malaysia's formation of lunar crescent visibility criterion in 1983. Turkey's 1978 lunar crescent visibility criterion is the result of an international conference in 1978. The conference was attended by representatives from 20 countries, including Malaysia and Indonesia. The purpose of the conference was to coordinate the determination of the new Hijri month among Muslim countries. Through the conference a number of resolutions were produced, among them is a resolution on the Turkey 1978 lunar crescent visibility criterion that had been mutually agreed upon by the representative. The criterion that has been agreed upon are as follows. The new Hijri month begins when a lunar crescent:

a. Has elongation parameter of more than 8 degrees; and
b. Has altitude of more than 5 degrees.

In 2016, Turkey proposed yet another lunar crescent visibility criterion. The proposal was through the Conference of Islamic Calendar in Istanbul, Turkey 2016M/1437H (Rodzali & Man, 2021). The conference representatives voted and resolved that;

a. The entire world is to be seen as one union where the new Hijri month begins on the same day throughout the world.
b. A new Hijri month begins when in any part of the earth, theSun-Moon elongation at sunset reaches more than 8 degrees or more, and the altitude of the lunar crescent is 5 degrees above the horizon.

This criterion, henceforth known as the Istanbul 2016 criterion, acts as a baseline for International Lunar Dateline. However, the Istanbul 2016 criterion seems to ignore a number of lunar crescent observation records. The world records for elongation are 7.7 degrees at naked eye, 6.0 degrees at optical aided, 6.8 degrees at telescopic observation, and 3.42 degrees at CCD imaging.(ICOP, 2022). The world records for altitude are 4.06 degrees at naked eye, 6.48 degrees at optical aided, 4.81 degrees at telescopic observation, and 4.62 degree at CCD imaging.

In 2017, Indonesia suggested another criterion known as Jakarta Recommendation 2017 (Sopwan & Al-Hamidy, 2020). The criterion acts as a supplement for the Istanbul 2016 criterion. The resolution on the criterion parameter was conducted through a discussion during a conference known as the "International Seminar on Astronomical Fiqh Opportunities and Challenges Implementation of the Single Hijri Calendar ", held in Jakarta on 29 to 30 November 2017. The conference was attended by participants from five countries, namely Indonesia, Jordan, Malaysia, Singapore, and Brunei Darussalam. This led to the formation of the Jakarta Recommendation 2017. The Jakarta Recommendation 2017 can be summarised as follows;

a. The Sun-Moon elongation at sunset reaches more than or equal to 6.4 degrees, and
b. Altitude of the lunar crescent during sunset is more or equal to 3 degrees above the horizon.

The Jakarta Recommendation 2017 is more suited for Hijri calendar determination, as it follows the International Crescent Observation Project world sighting records.

MABIMS's Lunar Crescent Visibility Criterion

Malaysia, Indonesia, Brunei and Singapore employed lunar crescent visibility criterion to determine their Hijri month (Azhari, 2021). These countries independently determined their own date of the first day of *Ramadan* and *Shawal,* at the same time they collaborate in formulating lunar crescent visibility criterion to determine the first date of the other Hijri months (Nawawi et al., 2015). In conjunction with the Association of Southeast Asian Nations (ASEAN), these four countries conjoined to form a governing body known as MABIMS (The Informal Meeting of Religious Ministries of Malaysia, Indonesia, Brunei and Singapore). The role of MABIMS is monitor the laws and principles on lunar crescent sighting and its visibility criterion to ensure that there is no disagreement among the members (Wahidi et al., 2019).

As an intercessor for these four countries, each of them is portrayed by their lunar calendar governing bodies: the Department of Islamic Development Malaysia (JAKIM) for Malaysia, the Ministry of Religious Affairs, Republic of Indonesia (KEMENAG RI) for Indonesia, the Islamic Religious Council of Singapore (MUIS) for Singapore and the Brunei Islamic Religious Council (MUIB) for Brunei. In summary, JAKIM, KEMENAG RI, MUIS, and MUIB are the governing authorities who are responsible for determining the dates of the Hijri calendar.

In 1995, Malaysia, Indonesia, Brunei and Singapore adopted the criterion for lunar crescent visibility known as as "*Imkan al-Rukyah*", which defines the beginning of the lunar months as "when the lunar crescent could be visible against clear skies" (Mamat, 2020). The concept of this criterion is the "possibility of visibility" which is based on the result of the visible crescent that has been sighted" - by using this lunar crescent visibility criterion (Azhari, 2012), the lunar crescent is expected to be seen when it fulfils one of the following conditions:

a. During sunset, the sun-moon elongation reaches more or equal to 3 degrees and the altitude of the lunar crescent reaches more or equal to 2 degrees above the horizon; or,
b. During the moonset, the moon age is more or equal to 8 hours.

The criterion suggests that if observation of a lunar crescent at a certain 29th Hijri day shows that it has altitude and elongation of more than 2 degrees and 3 degrees, respectively, then the next day is commenced as a new Hijri month. Based on the presence of the lunar crescent in Indonesia from the 1960s to the 1990s, the lunar crescent was reported to have appeared several times at an altitude of 2 degrees and elongation of 3 degrees. This criterion was then formulated based on the presence of the lunar crescent which was confirmed by KEMENAG RI at the time (JAKIM, 1991).

The lunar crescent altitude, elongation, and moon age parameter used in MABIMS lunar crescent visibility criterion were found be to conflicting with other research findings. Elongation criterion have been found by Schafer, Ilyas, Fatoohi and Odeh to be above 7 degrees for naked eye observation, and no lunar crescent was able to be sighted at an elongation of below 7 degrees, except for extreme optical aided observation. The same goes for the moon age and lunar crescent altitude, where the world records are 14 hours and 4 degrees for moon age and arc of vision respectively, a parameter that is significantly higher than MABIMS 1995 criterion. This indicates that the current lunar crescent visibility criterion adopted by MABIMS is outdated, without any current scientific evidence, and is not supported among lunar crescent visibility researchers.

In 2021, Malaysia, Indonesia, Brunei, and Singapore adopted a new criterion for lunar crescent visibility known as the Neo MABIMS Criteria. This criterion is the culmination of research and discussion among researchers, government officials, observatories, and universities in improving the previously flawed 1995 lunar crescent visibility criterion. The new criterion negates the use of moon age parameters, since it has been proven to be ineffective in finding lunar crescent visibility (N. Ahmad et al., 2020, 10; Alrefay et al., 2018,12; Anwar et al., 2016, 4; Ilyas, 1983, 1). The new criterion is formed considering the elongation parameter of Odeh lunar crescent visibility criterion (Odeh, 2004, 11), with additional MABIMS' own altitude parameter, originating from Jakarta Recommendation lunar crescent visibility criterion (Azhari, 2021, 11). The criterion is that the sun-moon elongation at sunset reaches more or equal to 6.4 degrees and altitude of the lunar crescent during sunset is more or equal to 3 degrees above the horizon.

4. Discussion and Conclusion

A review was conducted to demonstrate the scientific outlook of a lunar crescent visibility criterion: data locality, prediction of strengths and weaknesses, and its long-term legacy in visibility. The reviews demonstrate that each lunar crescent visibility criterion has its own strengths, limitations and application for calendrical determination and successful observation. From the reviews, there are a number of factors that cause the heterogeneity of a lunar crescent visibility criterion.

First, there is the differences of mathematical model. Caldwell used the lagtime parameter as the main variable in the criteria to show the existence of the influence of the geocentric model (Nawawi et al., 2012). Modern astronomers are more inclined towards mathematical models which are topocentric, that is by using the altitude, azimuth and elongation parameters change according to the position of the observer on the earth's surface. This parameter is used by the majority of experts who study the criteria such as Ilyas, McNally and Fotheringham. The differences in terms of the mathematical model used will impact the criterion construction. This is because different mathematical models will be used for different parameters and subsequently provide heterogeneity in the criterion. This demonstrates the influence of mathematical models on the construction of criterion.

Next, there is the primary concept of the criterion designer. Each lunar crescent visibility criterion is built based on one primary concept that is produced by the researcher himself. This concept was derived based on the background of society and culture, which influences the researcher's thinking and motivation when producing the criteria. A key example of how a concept can influence a lunar crescent visibility criterion can be seen from Ilyas's lunar crescent visibility criterion. Ilyas is an atmosphere astronomer from Universiti Sains Malaysia. Most of the criteria produced by him are much significantly higher than other criteria of lunar crescent visibility. Researchers such as Fatoohi and Schaefer found that lunar crescent located above his criterion is easily detected by the naked eye. The construction of a high visibility criteria for the moon, which can usually be seen by the human eye, will facilitate the production of calendars for large lines of longitude. The construction of criteria for countries of great longitude is very difficult because the rate of lunar crescent visibility will decrease when going east. This demonstrates that it can influence the concept of the Universal Islamic Calendar to Ilyas lunar crescent visibility criterion.

Another factor for lunar crescent criterion dissimilarity is the preference on the type of visibility. There are times when the lunar crescent is easily spotted by the naked eye. There are also times when the lunar crescent is vaguely visible and can only be tracked by using a telescope. This results in major complications in the criteria construction because each researcher has different visibility preference in building visibility criteria. Some have built criteria based on the conditions that it is easily visible to the naked eye, and there are also those who construct criteria based on its telescopic visibility. Another example of how selection on the range of visibility can affect the results of the lunar crescent visibility criterion is exemplified by Fatoohi in 1998. This shows that the selection of the range of visibility of the moon, either easily seen with the naked eye, or can only be seen using a telescope affects how a crescent visibility criterion is constructed. An example is the difference in approach by Fotheringham and Maunder in the construction of their criteria. Lunar crescent visibility criterion expressed by Fotheringham is different from the criterion by Maunder, although both of them used almost the same data in their respective studies. This difference occurs because of the criterion built by Fotheringam based on naked eye visibility of the lunar crescent. When the parameter of the moon passes the conditions specified by Fotheringham, the crescent moon is definitely visible to the naked eye. Maunder, on the other hand, preferred critical limit of visibility between positive and negative naked eye sightings. Therefore, a lunar crescent located above Maunder limits is easily visible by telescope, but is not necessarily visible to the naked eye.

The final factor of criteria dissimilarity is the differences in data used for criterion construction. The lunar crescent visibility criterion is built based on empirical data collected by past researchers. There are researchers who have a large collection of lunar crescent sightings, and there are also researchers who build their criteria based on a limited collection of lunar crescent sightings. The amount of data on lunar crescent sightings and the distribuition of the data influences the graph and criterion of a lunar crescent sighting. An excellent example can be seen in the results of a study conducted by Yallop, Odeh, and Qureshi. The criteria built by Yallop, Odeh, and Qureshi used the same concept, which is the construction of criteria that take into account various ranges of visibility, including visibility with a telescope, visibility to the naked eye, and visibility with binoculars. Theyalso used the same model, which is the topocentric model, taking into account the width of the crescent moon as the main parameter. However, the Yallop, Odeh, and Qureshi criteria are different from each other. This is due to the differences of reference data on lunar crescent sightings between Yallop, Odeh and Qureshi. Yallop has 295 data, Odeh has 737 data, while Qureshi has 436 data. The differences in data reference and the distribution of the data makes the criteria produced by Yallop, Odeh and Qureshi differ from each other even though their criteria are constructed using the same mathematical concepts and models.

5. Acknowledgements

This research is supported by the University of Malaya Grant, under project number of UMG004L-2022 and RMF0068-2021.

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