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A NUMERICAL MODEL FOR THE SIMULATION OF DOUBLE-DIFFUSIVE NATURAL CONVECTION IN A RIGHT-ANGLED TRIANGULAR SOLAR COLLECTOR

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ABSTRACT
A numerical model is presented for the simulation of double-diffusive natural convection in a triangular solar collector. This design is encountered in greenhouse solar stills where vertical temperature and concentration gradients between the saline water and transparent cover induce flows in a confined space. This phenomenon plays an important function in the water distillation process and in the biological comfort. In this double-diffusion problem, the ratio *Br* of the relative magnitude thermal and compositional buoyancy and Rayleigh numbers are key parameters. Finite element technique is used to solve the governing equations. Numerical results are presented for the effect of the abovementioned parameters on local heat and mass transfer rate. In addition, results for the average heat and mass transfer rate are offered and discussed for the mentioned parametric conditions. Some interesting results are found in this investigation.

Keywords: Solar collector, Double-diffusion, Finite element method, Saline water, Transparent cover.

1. INTRODUCTION

Natural convection flow and heat transfer in different geometrical enclosures are the key topic of enormous research engineering studies. These studies consist of various scientific applications such as solar collectors, building heating and ventilation, cooling electrical devices introduced by Gebhart et al (1988), De Vahl Davis and Jones (1983), and Ostrach (1988). Double-diffusive natural convection occurs within the solar collector, due to the combined thermal and mass diffusion buoyancy effects as well as the temperature difference between cover and absorber. The study of solar collectors is one of the most significant mechanisms to progress their performance with a competitive price in recent years. Besides, numerical investigation has the advantage over an experimental study. The vital parameters, namely glass thickness, covenant location, and geometrical dimensions may be simply changed. Therefore, it's persuading on the overall heat and mass transfer may be analyzed at a low cost. Numerical exploration is also supportive in testing the performance of solar collectors for different components. Numerous experimental and theoretical investigations have been conducted in this area. The performance of one-sided vertical solar still tested under desert climatic conditions of Algeria, in summer and autumn seasons 2003, have been evaluated by Boukar and Harmim (2005). Later, the same authors experimentally studied the design parameters of an indirect vertical solar still (2007). The thermal exchange by natural convection and effect of buoyancy force on flow structure was examined by Omri et al. (2005). Subsequently, Omri (2007) made a numerical model to investigate the flow characteristics inside an asymmetrical triangular still for the configuration optimization. At the same time, another numerical experiment is performed for inclined solar collectors by Varol and Oztop (2007). A little number of mathematical models of the energy balance equations describing the heat and mass transfer in a solar still has been presented. The chiefly envoys of such studies are those projected by Selcuk (1971) and Sodha et al (1981). To investigate the performance of a prism shaped storage solar collector with a right triangular cross sectional area, Joudi et al. (2004) offered a mathematical model. Different types of solar still available in the literature are conventional solar stills, single-slope solar still with passive condenser, double condensing chamber solar still in Tiwari et al. (1997), vertical solar still in Coffey (1975), the inverted absorbers solar still in Suneja and Tiwari (1999), and multiple effect solar still in Tanaka et al. (2000).

The review given above shows that the double diffusive natural convection in triangular types of solar collectors has not been studied yet. Thus, the chief aim of the present study is to present a mathematical model to investigate the effect on natural convection heat and mass transfer in a triangular shaped enclosure for different buoyancy ratio and Rayleigh numbers.

2. PROBLEM STATEMENT

2.1 *Physical Model*

The schematic configuration of a two-dimensional triangular type solar collector is shown in Fig. 1. The enclosed space consists mostly of an absorber plate and two inclined glass covers that form an enclosure. In addition, the absorber plate is represented by a horizontal bottom wall kept at a higher fixed temperature T_h and concentration c_h. The inclined and vertical walls are considered transparent and maintained a steady temperature T_L and concentration c_L with $T_h > T_L$ and also $c_h > c_L$.

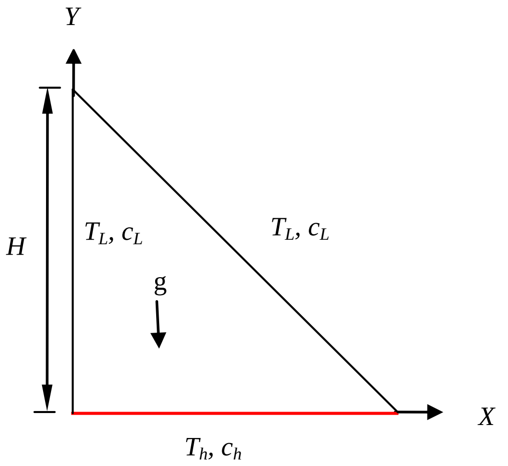

Fig. 1 Schematic diagram for the problem

2.2 *Mathematical Model:*

In this study, we consider the salty water on the bottom side of the glazing enclosure which is vaporized from the liquid-vapor boundary. It is clear that the vapor moves through the air and condenses at the cooled side walls. Here, the conservation equations have been expressed using the velocity-pressure formulation assuming constant properties except for the density in the body force term. The steady two-dimensional laminar convection the coupled transport governing equations for U, V, θ and C can be written in the dimensionless form as:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+ Ra_T Pr\ \ \theta+BrC \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2} \tag{4}$$

$$U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{Le}\left(\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2}\right) \tag{5}$$

The dimensionless variables used in the above equations are introduced as:

$$X=\frac{x}{H}, Y=\frac{Y}{H}, U=\frac{uH}{\nu}, V=\frac{vH}{\nu}, \quad P=\frac{p+\rho gy\ \ H^2}{\rho\nu^2}, \theta=\frac{T-T_L}{T_h-T_L}, \text{ and } C=\frac{c-c_L}{c_h-c_L} \tag{6}$$

The dimensionless parameters appearing in the equations (2) – (5) are the Prandtl number (Pr), the thermal Rayleigh number (Ra_T), Lewis number (Le) and buoyancy ratio (Br), which can be expressed as:

$$Pr=\frac{\nu}{\alpha}, Ra_T=\frac{g\beta_T\ \ T_h-T_L\ \ H^3}{\alpha\nu}, \quad Le=\frac{\alpha}{D}, \text{ and } Br=\frac{\beta_c\ \ c_h-c_L}{\beta_T\ \ T_h-T_L} \tag{7}$$

The dimensionless boundary conditions corresponding to the present study are: at the bottom horizontal wall: $U=V=0, \theta=C=1$

at the inclined and vertical walls: $U=V=0, \theta=C=0$

The local heat and mass transfer rates on the surface of heat and contaminant sources are defined respectively as

$$Nu_X=-\left.\frac{\partial\theta}{\partial Y}\right|_{Y=0} \text{ and } Sh_X=-\left.\frac{\partial C}{\partial Y}\right|_{Y=0}$$

The average heat and mass transfer rates on the surface of heat and contaminant sources can be evaluated by the average Nusselt and Sherwood numbers, which are defined respectively as

$$Nu=-\int_0^1\frac{\partial\theta}{\partial Y}dX \text{ and } Sh=-\int_0^1\frac{\partial C}{\partial Y}dX$$

The stream function is calculated from

$$U=\frac{\partial\psi}{\partial Y},\ V=-\frac{\partial\psi}{\partial X}$$

3. NUMERICAL TECHINIQE

In this study, the Galerkin weighted residual of finite element method has been applied as a numerical procedure. The finite element method starts by the partition of the continuum area of interest into a number of simply shaped regions known as elements. These elements may be different shapes and sizes. The dependent variables are approximated using interpolation functions for each element. The erratic grid size system is considered especially near the walls to capture the rapid changes in the dependent variables. The coupled governing equations (2)-(5) are transformed into sets of algebraic equations using finite element method to reduce the continuum domain into discrete triangular domains. The system of algebraic equations has been solved by iteration technique. The solution process is iterated until the subsequent convergence condition is satisfied: $\left|\Gamma^{m+1}-\Gamma^{m}\right|\le 10^{-6}$ where m is number of iteration and Γ is the general dependent variable.

The study is compared with an earlier work on natural convection in a triangular enclosure introduced by Kent (2009). Comparisonal results are illustrated in Fig. 2 for different inclination angle which is studied by Kent (2009). The authour used Fluent commercial code as numerical analysis. The comparisonal results showed a good agreement with literature on heat transfer and fluid flow. Then, we extened and add mass transfer to simulate solar collectors.

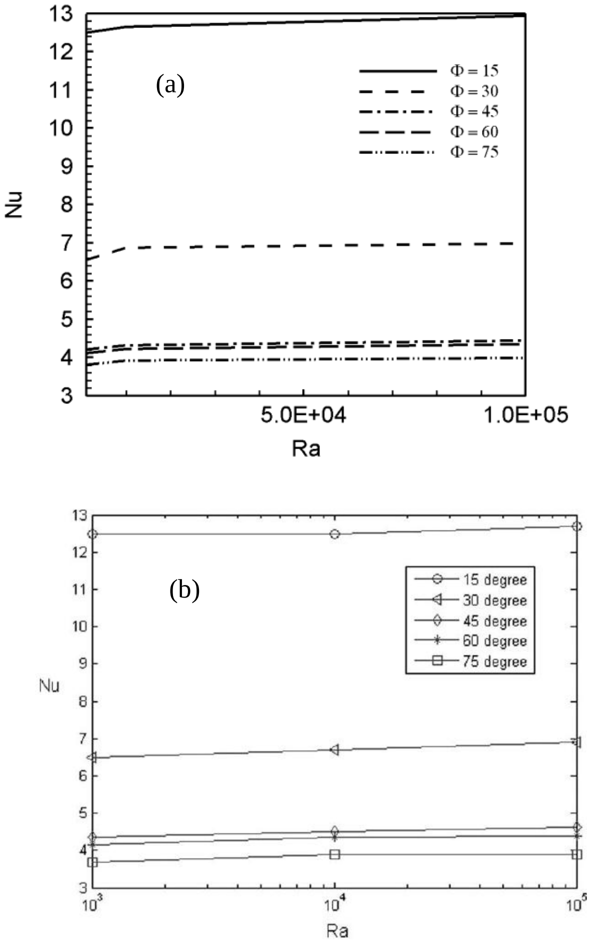

Fig. 2 Comparison of the (a) present model with (b) the results of Kent (2009) for natural convection inside a isosceles triangular enclosure for five different base angles.

4. RESULTS AND DISCUSSION

A numerical study has been performed through finite element method to analyze the laminar natural convection heat transfer and fluid flow inside an isosceles solar collector. Water is considered as working fluid inside the enclosure having Pr = 7. In addition, the Lewis number kept fixed at Le = 2 throughout this investigation. Effective parameters such as Rayleigh number and buoyancy ratio, on heat transfer and fluid flow are investigated. The distribution of local heat transfer rate along the bottom wall has been depicted in Fig. 3 at different Rayleigh numbers. If we look at the variation of average heat transfer rate with the buoyancy ratio, it gives increasing trend with buoyancy ratio and Rayleigh number. It is an interesting result that heat transfer goes to minimum at X = 0.5 for all parameters. The reason of this value is the stagnation point and minimum velocity at this point. Trend of local heat transfer rate exhibits a symmetric distribution according to the mid-axis. As an expected result, highest heat transfer is formed for Ra = 10^5 as shown in Fig. 3(c). One may notice that the heat transfer rate decreases from the left and right edge to the mid-axis. It is also observed that it has maximum values around X = 0.25 and 0.85 for higher values of the buoyancy ratio.

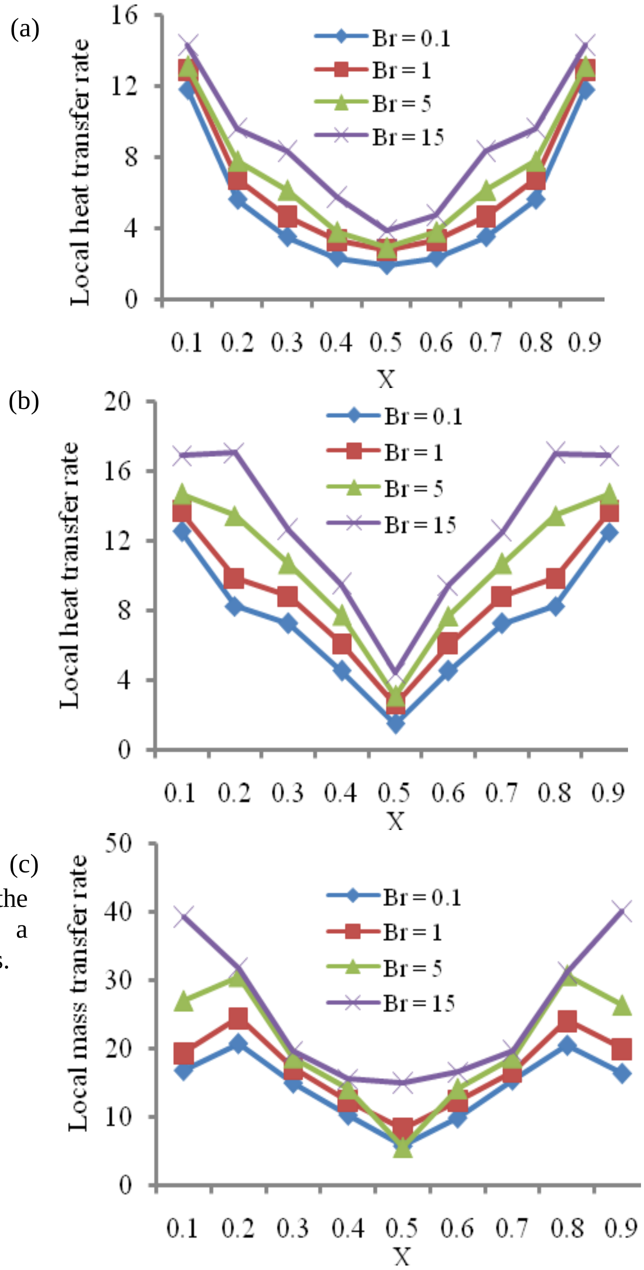

Fig.3. Effect of buoyancy ratio on local heat transfer rate at (a) $Ra = 10^3$, (b) $Ra = 10^4$ and (c) $Ra = 10^5$.

The effect of buoyancy ratio on local mass transfer rate is given in Fig. 4 at different Rayleigh numbers. Its trend is almost similar with the local heat transfer rate, and it decreases with decreasing of the buoyancy ratio for all *Ra*. The highest value is formed at Ra = 10^5 and Br = 15 around X = 0.25 and 0.85.

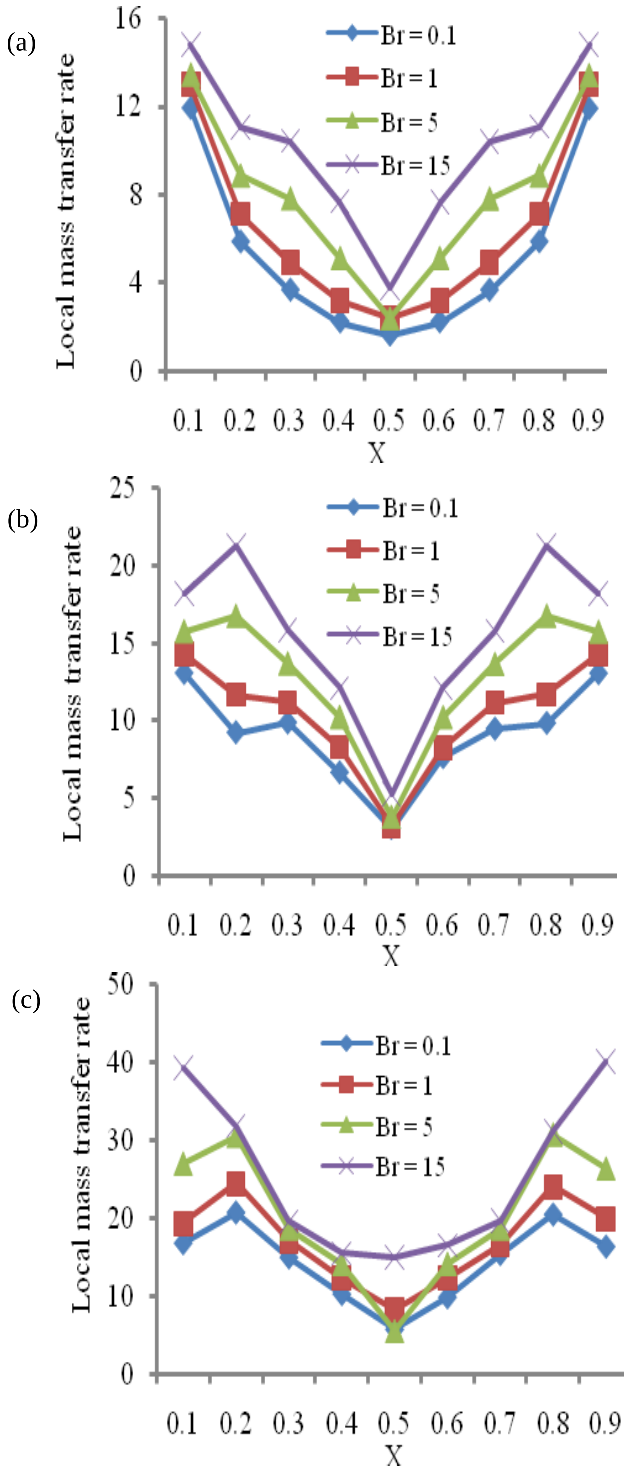

Fig. 4. Effect of buoyancy ratio on local mass transfer rate at (a) $Ra = 10^3$, (b) $Ra = 10^4$ and (c) $Ra = 10^5$.

The effect of buoyancy ratio on average heat and mass transfer rate for considered Rayleigh numbers is shown in Fig. 5. From this figure it is evident that a linear increasing is obtained according to the buoyancy ratio. Average value of mass transfer rate increases with increasing *Ra*.

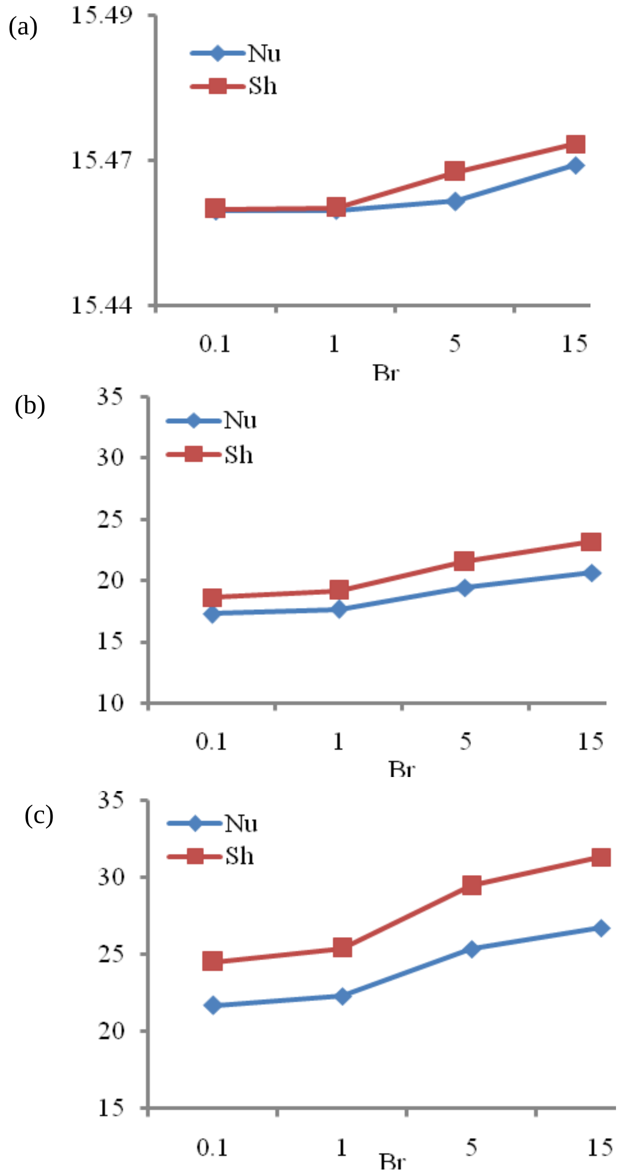

Fig. 5. Effect of buoyancy ratio on average heat and mass transfer rate at (a) $Ra = 10^3$, (b) $Ra = 10^4$ and (c) $Ra = 10^5$.

5. Conclusions

The heat transfer and mass transfer enhancement inside triangular shaped solar collectors has been investigated numerically. The results are displayed for different values of buoyancy ratio and Rayleigh number. Some vital observations may be listed as follows:

(i) Local heat transfer and mass transfer have maximum value near the left and also right of the bottom wall.

(ii) Both the heat transfer and mass transfer increase with increasing of buoyancy ratio as well as Rayleigh numbers.

(iii) Increasing of the buoyancy ratio for the same Rayleigh number enhances the local heat and mass transfer rate.

NOMENCLATURE:

Br	buoyancy ratio
c	concentration of species
c_h	high species concentration (source)
c_L	low species concentration (sink)
C	high species concentration (source)
D	species diffusivity
g	gravitational acceleration
H	enclosure height
Le	Lewis number
Nu	average Nusselt number
p	dimensional pressure
P	non-dimensional pressure
Pr	Prandtl number
Ra_T	thermal Rayleigh number
Sh	average Sherwood number
T	temperature
T_h	hot wall temperature (source)
T_L	cold wall temperature (sink)
u	horizontal velocity component
U	dimensionless horizontal velocity component
v	vertical velocity component
V	dimensionless vertical velocity component
x	horizontal coordinate
X	dimensionless horizontal coordinate
y	vertical coordinate
Y	dimensionless vertical coordinate

Greek symbols

α	thermal diffusivity
β_T	thermal expansion coefficient
β_c	compositional expansion coefficient
ν	kinematic viscosity
θ	non-dimensional temperature
ρ	density
Ψ	stream function

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